

BEARINGS

A bearing is a mechanical element that permits relative motion between two parts, such as the shaft and the housing, with minimum friction.

The functions of the bearing are as follows:

- i) The bearing ensures free rotation of the shaft on the axle with minimum friction.
- ii) The bearing supports the shaft on the axle and holds it in correct position.
- iii) The bearing takes up the forces that act on the shaft on the axle and transmits them to the frame or the foundation.

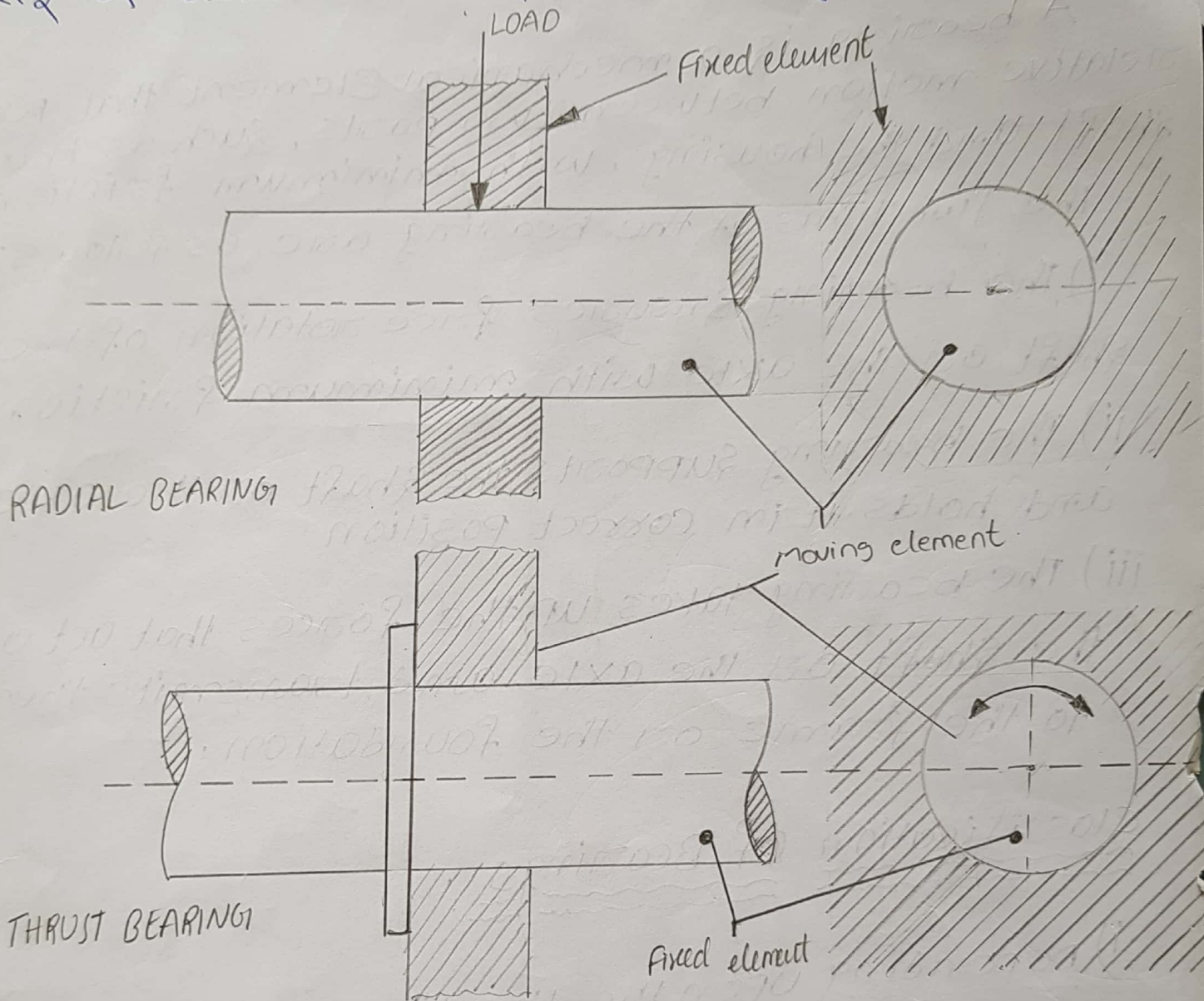
Classification of Bearings :

i) Depending upon the direction of load to be supported :

- a) Radial bearings
- b) Thrust bearings

In radial bearings, the load acts perpendicular to the direction of motion of moving element as shown Fig

In thrust bearings, the load acts along the axis of rotation as shown in Fig.



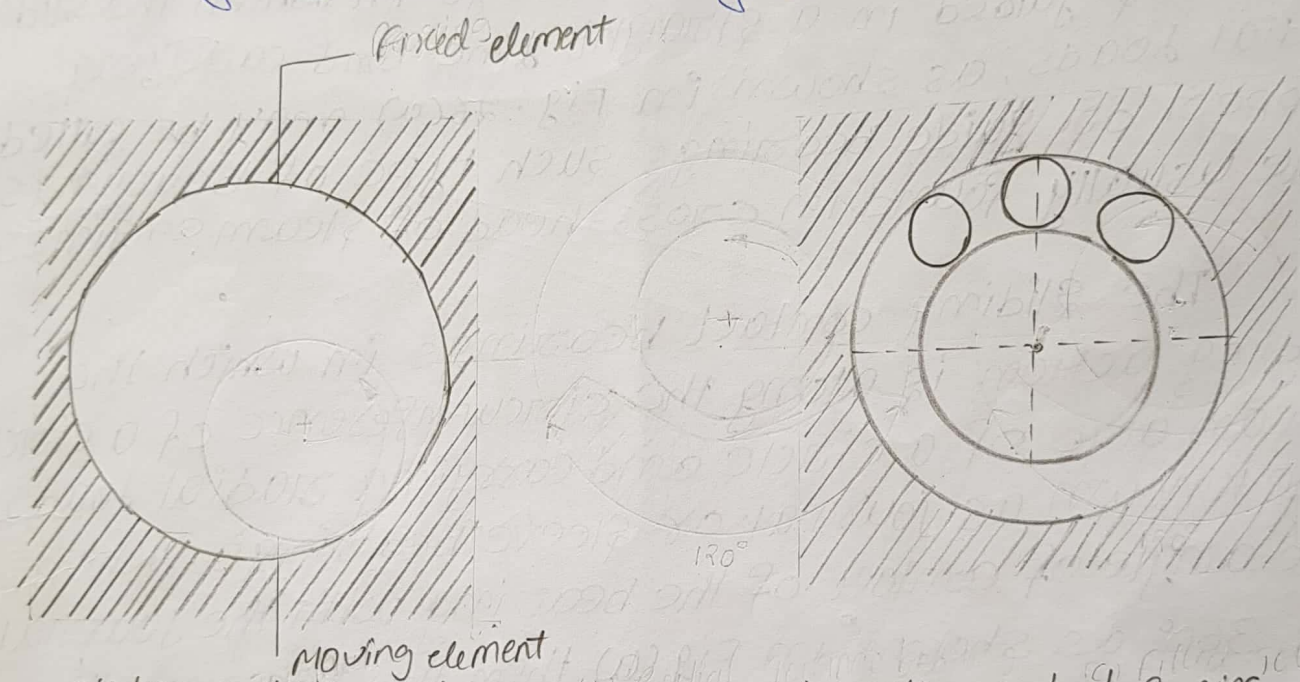
2. Depending upon the nature of contact:

- a) Sliding contact bearings
- b) Rolling contact bearings

In sliding contact bearings as shown in Figure, the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as plain bearings.

may be called as clearance bearings because the diameter of the journal is less than that of bearing.

When a partial journal bearing has no clearance i.e. the diameters of the journal and the bearing are equal, then the bearing is called a fitted bearing as shown in Fig (c).



(a) Sliding Contact Bearing

(b) Rolling Contact Bearing

The sliding contact bearings, according to the thickness of layer lubricant between the bearing and the journal, may also be classified as follows.

1. Thick film bearings : The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as hydrodynamic lubricated bearings.

In rolling contact bearings as shown in Fig, the steel balls or rollers are interposed between the moving and fixed elements. The ball offers rolling friction at two points for each ball or roller.

Types of Sliding Contact Bearings :

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, as shown in Fig. 26(a) may be called slipper or guide bearings such type of bearings are usually found in cross-head of steam engines.

The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as journal or sleeve bearings. When the angle of contact of the bearing with the journal is 360° as shown in Fig (a) then the bearing is called full journal bearing. This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction.

When the angle of contact of the bearing with the journal is 120° , as shown in Fig (b) then the bearing is said to be partial journal bearing. This type of bearing has less friction than full journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings

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i) Deep Groove Ball bearing: The most frequently used bearing is deep groove ball bearing. It is found in almost all kinds of products in general mechanical Engineering. In this type of bearing, the radius of the ball is slightly less than the radii of curvature of the grooves in the races. Kinematically this gives a point of contact between the balls and the races. Therefore the balls and the races may roll freely without any sliding.

ii) Cylindrical Roller Bearing: When maximum load carrying capacity is required in a given space the point contact in ball bearing is replaced by the line contact of roller bearing. It consists of relatively short rollers that are positioned and guided by the cage.

iii) Angular contact Bearing: In angular contact bearing, the grooves in inner and outer races are so shaped, that the line of reaction at the contact between balls and races makes an angle with the axis of the bearing. This reaction has two components radial and axial therefore it can take radial and thrust loads.

Thin film bearings: The thin film bearings are those in which although lubricant is present the working surfaces partially contact each other at least part of the time. Such type of bearings are also called boundary lubricated bearings.

3. Zero film bearings: The zero film bearings are those which operate without any lubricant present.

4. Hydrostatic or externally pressurized lubricated bearings.

The hydrostatic bearing are those which can support steady loads without any relative motion between journal and the bearing. This is achieved by forcing externally lubricant between the members.

Rolling contact bearings:

For starting conditions and at moderate speeds, the frictional losses in rolling contact bearing are lower than that of an equivalent hydrodynamic journal bearing. This is because the sliding contact is replaced by rolling contact resulting in low coefficient of friction. Therefore, rolling contact bearings are called antifriction bearings.

The type of rolling contact bearing which are frequently used.

Static Load Carrying Capacity: Static load is defined as the load acting on the bearing when the shaft is stationary. It produces permanent deformation in balls and races which increases with increasing load. The permissible static load, therefore, depends upon the permissible magnitude of permanent deformation.

It has been found that a total deformation of 0.0001 of the ball or roller diameter occurring at the most heavily stressed ball and race contact, can be tolerated in practice, without any disturbance like noise or vibrations. The static load carrying capacity of a bearing is therefore defined as the static load which corresponds to a total permanent deformation of balls and races at the most heavily stressed point of contact, equal to 0.0001 of the ball diameter.

Dynamic Load Carrying Capacity: The life of a ball bearing is limited by the fatigue failure at the surfaces of balls and races. The dynamic load carrying capacity of the bearing is therefore based on the fatigue life of the bearing.

The life of an individual ball bearing is defined as the number of revolutions (or hours of service at some given constant speed), that the bearing runs before the first evidence of fatigue crack in balls or races.

The rating life of a group of apparently identical ball bearings is defined as the number of revolutions that 90% of the bearings will complete or exceed before the first evidence of fatigue crack.

4) Self-Aligning Bearings: These are two types of self-aligning rolling contact bearings, i.e. self-aligning ball bearing and spherical roller bearing [The principle of self-aligning]. The self-aligning ball bearing consists of two rows of balls, that roll on a common spherical surface in the outer race. In this case the assembly of the shaft, the inner race and the balls with cage can freely roll and adjust itself to the angular misalignment of the shaft.

There is similar arrangement in spherical roller bearing, where balls are replaced by two rows of spherical rollers that run on a common spherical surface in the outer race.

v) Taper Roller Bearing: The taper roller bearing consists of rolling elements in the form of frustum of a cone. They are arranged in such a way that the axes of individual rolling elements intersect in a common apex point on the axis of the bearing. In kinematics analysis this is the essential requirement for pure rolling motion between conical surfaces.

vi) Thrust Ball Bearing: Thrust ball bearing consists of a row of balls running between two rings - the shaft ring and the housing ring. Thrust ball bearing carry thrust load in only one direction and cannot carry any radial load.

There are number of terms used for this rating life. They are minimum life, catalogue life, L_{10} life or B_{10} life.

The life of an individual ball bearing may be different from rating life. Statistically it can be proved that the life, which 50% of a group of bearings will complete or exceed, is approximately five times the rating or L_{10} life. This means that for majority of bearings the actual life is considerably more than the rated life.

The dynamic load carrying capacity of a bearing is defined as the radial load in radial bearings (or thrust load in thrust bearings) that can be carried for a minimum life of one million revolutions. The minimum life [of one million revolutions] in this definition is the L_{10} life that 90% of the bearings will reach or exceed before fatigue failure.

Equivalent Bearing Load: The equivalent dynamic load is defined as the constant radial load in radial bearings (or thrust load in thrust bearings) that if applied to the bearing would give same life as that which the bearing will attain under actual condition of forces. The expressions for the equivalent dynamic load is written as,

$$P = (XV F_r + Y F_a)$$

Where

P = Equivalent dynamic load (N)

F_r = Radial load (N)

F_a = axial or thrust load (N)

V = Vane - rotation factor

S = Service factor

X & Y are radial and thrust factors respectively and their values are given in the manufacturer's catalogues. Assuming V as unity, the general Equation for equivalent dynamic load is given by

$$P = XF_r + YF_a$$

* When the bearing is subjected to pure radial load
i.e. $P = F_r$

* When the bearing is subjected to pure thrust load
i.e. $P = F_a$

Load Life Relationship:

The relationship between the dynamic load carrying capacity, the equivalent dynamic load and the bearing life is given by

$$L_{10} = \left(\frac{C}{P} \right)^k$$

Where

L_{10} = rated bearing life (in million revolutions)

C = dynamic load capacity (N)

k = 3 (for ball bearings)

= $\frac{10}{3}$ (for roller bearings)

From the above equation

$$C = P(L_{10})^{1/k}$$

(11)
* For all type of ball bearings

$$C = P(L_{10})^{1/3}$$

* For all type of roller bearings

$$C = P(L_{10})^{0.3}$$

The relationship between life in million revolutions and life in working hours is given by

$$L_{10} = \frac{60 n L_{10h}}{10^6}$$

where

L_{10h} = rated bearing (hours)

n = speed of rotation

Selection of bearing from manufacturer's catalogue:

Step 1: Calculate the radial and axial forces acting on the bearing and determine the diameter of the shaft where the bearing is to be fitted.

Step 2: Select the type of bearing for the given application.

Step 3: Determine the values of X & Y , the radial and thrust factors from the catalogue. The values of X & Y factors for single-row deep groove ball bearings are taken from data book. The values depends upon two ratios $\left(\frac{F_a}{F_r}\right)$ and $\left(\frac{F_a}{C_0}\right)$.

C_0 = static load capacity

Note: The required data is taken from data book.

Step 4 : calculate the equivalent dynamic load from the equation.

$$P = XF_r + YF_a$$

Step 5 : Make decision about the expected bearing life and express the life L_{10} in million revolutions

Step 6 : Calculate the dynamic load capacity from the equation

$$C = P(L_{10})^{1/3}$$

Step 7 : Check whether the selected bearing of series 60 has the required dynamic capacity. If not select the bearing of the next series and go back to step (3) and continue.

Design for cyclic loads and speeds :

In certain applications, ball bearings are subjected to cyclic loads and speeds. As an example consider a ball bearing operating under the following conditions.

- i) radial load 2500 N at 700 rpm for 25% of the time
- ii) radial load 5000 N at 900 rpm for 50% of the time, and
- iii) radial load 1000 N at 150 rpm for the remaining 25% of the time.

Under these circumstances, it is necessary to consider the complete work cycle while finding out the dynamic load capacity of the bearing. The procedure consists of dividing work cycle into a number of elements, during which the operating conditions of load & speed are constant.

Suppose that the work cycle is divided into x Elements. Let $P_1, P_2, \dots, P_x$ be the loads and $n_1, n_2, \dots, n_x$ be the speeds during these elements. During the first element, the life L_1 corresponding to load P_1 is given by

$$L_1 = \left(\frac{C}{P_1} \right)^3 \times 10^6 \text{ million rev}$$

* In one revolution, the life consumed is $\left(\frac{1}{L_1} \right)$ or $\frac{P_1^3}{C^3} \times \frac{1}{10^6}$

Let us assume the first element consists of N_1 revolutions. Therefore the life consumed by the first element is given by

$$\frac{N_1 P_1^3}{10^6 C^3}$$

Similarly, the life consumed by the second element is given by

$$\frac{N_2 P_2^3}{10^6 C^3}$$

Adding these expression, the life consumed by the complete work cycle is given by

$$\frac{N_1 P_1^3}{10^6 C^3} + \frac{N_2 P_2^3}{10^6 C^3} + \dots + \frac{N_x P_x^3}{10^6 C^3} \quad \text{--- (1)}$$

If P_e is the equivalent load for the complete work cycle, the life consumed by the work cycle is given by

$$\frac{N P_e^3}{10^6 C^3} \quad \text{--- (2)}$$

where $N = N_1 + N_2 + \dots + N_x$

Evaluating Expressions (1) & (2)

$$N_1 P_1^3 + N_2 P_2^3 + \dots + N_x P_x^3 = N P_e^3$$

*

$$P_e = \sqrt[3]{\frac{N_1 P_1^3 + N_2 P_2^3 + \dots}{N_1 + N_2 + \dots}}$$

(or)

*

$$P_e = \sqrt[3]{\frac{\sum N P^3}{\sum N}}$$

Note: The above evaluation is used for calculating the dynamic load capacity of a bearing.

When the loads not vary in steps of constant magnitude, but varies continuously with time the above equation is modified and written as

*

$$P_e = \left[\frac{\int_0^N P^3 dN}{\int_0^N dN} \right]^{1/3}$$

or *

$$P_e = \left[\frac{1}{N} \int P^3 dN \right]^{1/3}$$

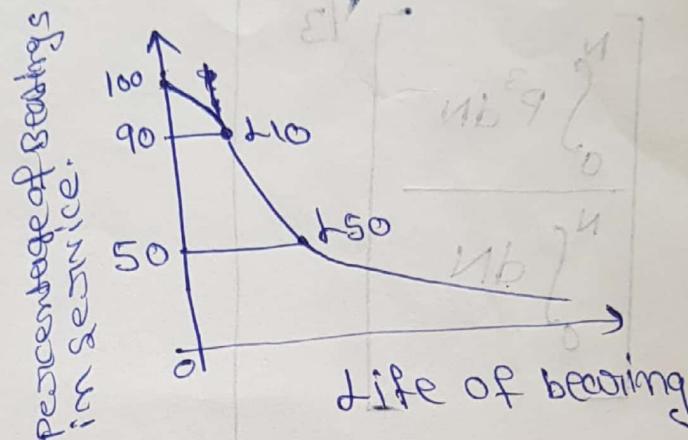
Bearing with a probability of survival other than 90%.

In the definition of rating life, it is mentioned that the rating life is the life that 90% of a group of identical bearings will complete or exceed before fatigue failure. The reliability R is defined as

$$R = \frac{\text{No. of bearings which have successfully completed 2 million revolutions}}{\text{Total number of bearings under test.}}$$

Therefore, the reliability of bearings selected from the manufacturer's catalogue is 0.9 or 90%.

The relationship between bearing life and reliability is given by a statistical curve known as Weibull distribution



For Weibull distribution

$$* R = e^{-(L/a)^b}$$

R = reliability (in fraction)

L = corresponding life

a, b = constants.

$$\frac{1}{R} = e^{(L/a)^b}$$

*

$$\log_e \left(\frac{1}{R} \right) = \left(\frac{L}{a} \right)^b \quad \text{--- (a)}$$

If L_{10} is the life corresponding to a reliability of 90% or R_{90} then

$$* \log_e \left(\frac{1}{R_{90}} \right) = \left(\frac{L_{10}}{a} \right)^b \quad \text{--- (b)}$$

dividing Eq (a) by Eq (b) we have

$$\left(\frac{d}{L_{10}} \right) = \left[\frac{\log_e \left(\frac{1}{R} \right)}{\log_e \left(\frac{1}{R_{90}} \right)} \right]^{1/b}$$

where $R_{90} = 0.9$

$a = 6.84$

$b = 1.17$

Sliding Contact Bearings :

Basic Modes of Lubrication :

Lubrication is the science of reducing friction by application of a suitable substance called lubricant, between the rubbing surfaces of bodies having relative motion. The lubricants are classified into following three groups:

- i) Liquid lubricants, like mineral or vegetable oils
- ii) Semi-solid lubricants, like grease
- iii) Solid lubricants, like graphite or molybdenum disulphide

The objectives of lubricant are as follows:

- i) to reduce friction;
- ii) to reduce or prevent wear;
- iii) to carry away heat generated due to friction and
- iv) to protect the journal and the bearing from corrosion

The basic modes of lubrication are thick and thin film lubrication. In addition, sometimes a term zero film bearing is used. Zero film bearing is a

bearing that operates without any lubricant i.e without any film of lubricating oil.

Thick film lubrication describes a condition of lubrication, where two surfaces of the bearing in relative motion are completely separated by a film of fluid. Since there is no contact between the surfaces the properties of surfaces, like surface finish, have little or no influence on the performance of the bearing. The resistance to relative motion arises from the viscous resistance of the fluid. Therefore the viscosity of the lubricant affects the performance of the bearing.

Thick film lubrication is further divided into two groups - hydrodynamic and hydrostatic lubrication.

hydrodynamic lubrication is defined as a system of lubrication in which the load supporting fluid film is created by the shape and relative motion of the sliding surfaces.

Principle of hydrodynamic lubrication in

journal bearing: Initially the shaft is at rest and it sinks to the bottom of the clearance space under the action of load W . The surfaces of the journal and bearing touch during rest. As the journal starts to rotate, it will climb the bearing surface and the speed is further increased, it will force the fluid into wedge-shaped region. Since more and more fluid is forced into the wedge-shaped clearance space, pressure is generated within the system.

The pressure distribution around the periphery of the journal: Since the pressure is created within the system due to rotation of the shaft, this type of bearing is known as self-acting bearing. The pressure generated in the clearance space supports the external load W . In this case, it is not necessary to supply the lubricant under pressure and the only requirement is sufficient and continuous supply of the lubricant. This mode of lubricant is seen in bearings mounted on engines and centrifugal pumps frequently, a term journal bearing is used.

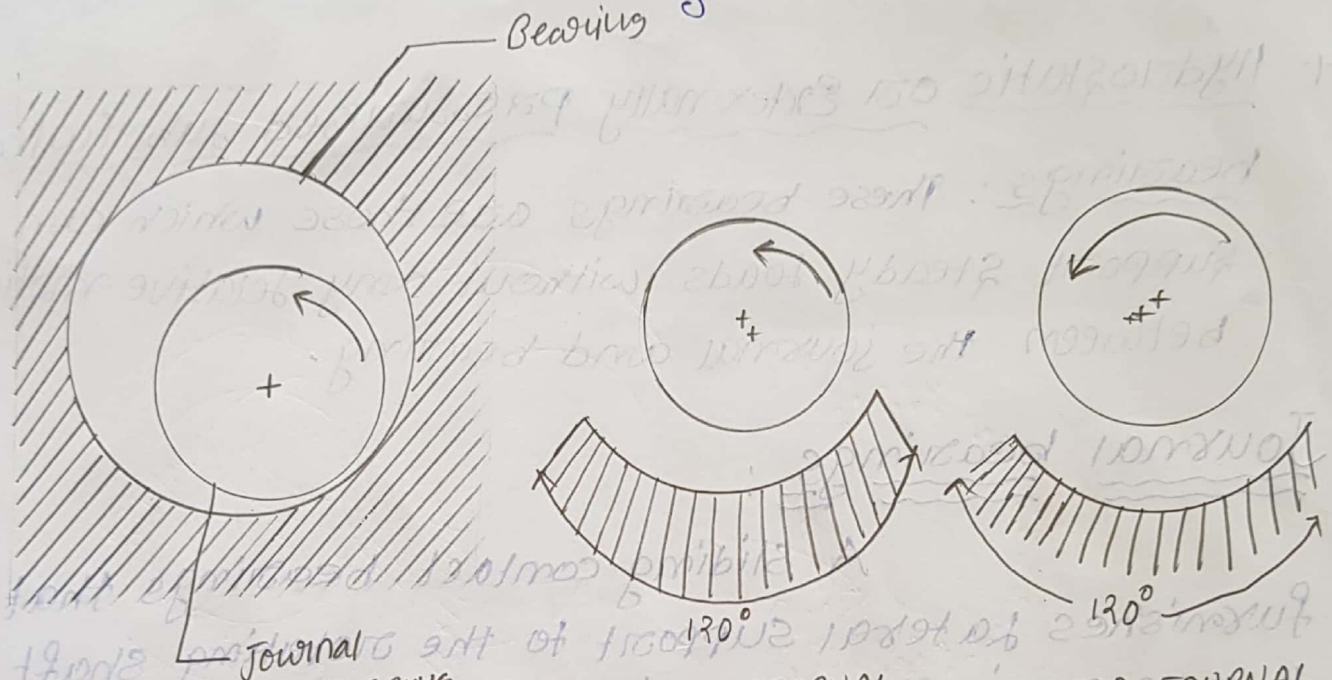
A journal bearing is a sliding contact bearing working on hydrodynamic lubrication and which supports the load in radial direction. The position of the shaft inside the bearing is called journal and hence the name journal bearing.

There are two types of hydrodynamic journal bearings namely, full journal bearing and partial bearing.

Full journal bearing: In full journal bearing the angle of contact of the bushing with journal is 360° . Full journal bearing can take load in any radial direction. Most of the bearings used in industrial applications are full journal bearing.

Partial bearing: In partial bearings the angle of contact between the bush and journal is always less than 180° . Most of the partial bearings in practice have 120° angle of contact. These bearings can take load in only one ^{radial} direction. These bearings are used for rail-road-cars.

fitted bearing : When a partial journal bearing has no clearance i.e. the diameters of journal and bearing are equal then the bearing is called a fitted bearing.



(a) FULL JOURNAL BEARING

(b) PARTIAL JOURNAL BEARING

(c) FITTED JOURNAL BEARING

The sliding contact bearings are also classified based on thickness of layer of the lubricant between the bearing and the journal. They are

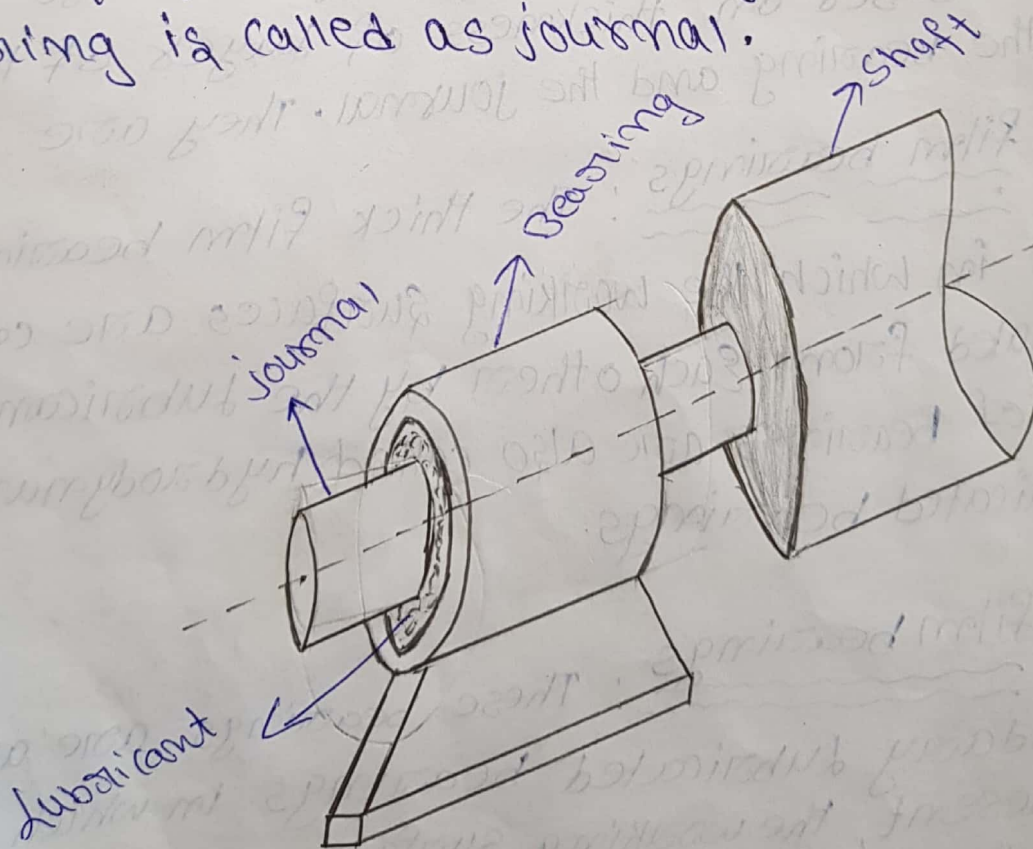
1. Thick film bearings : The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called hydrodynamic lubricated bearings.
2. Thin film bearings : These bearings are also called boundary lubricated bearings in which lubricant is present, the working surfaces partially contact each other at least part of the time.

3. Zero film bearings : The zero film bearings are those which operate without any lubricant present.

4. Hydrostatic or Externally Pressurized Lubricated bearings : These bearings are those which can support steady loads without any relative motion between the journal and bearing.

Journal bearings :

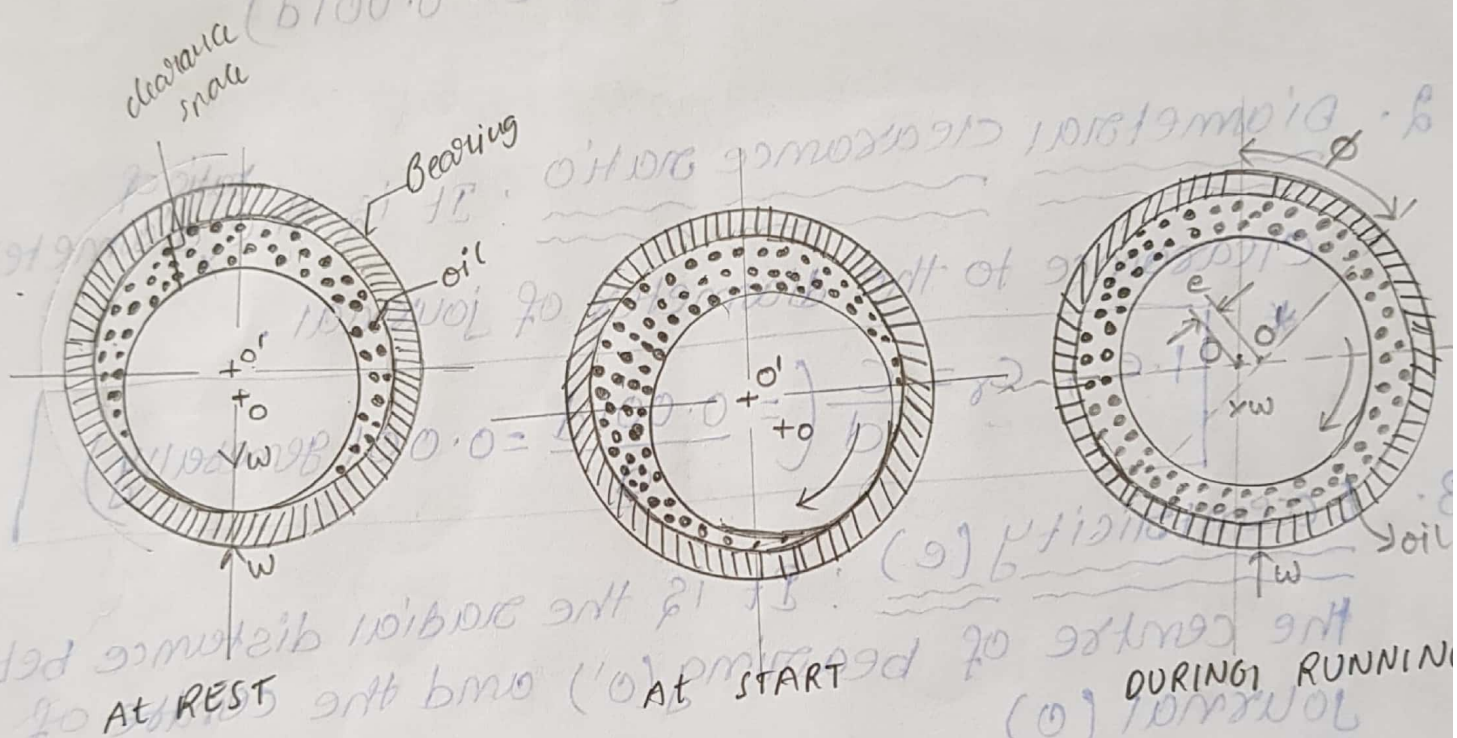
A sliding contact bearings that furnishes lateral support to the rotating shaft is known as journal bearing. It consists of two main parts, a shaft and a housing. The portion of the shaft inside the housing, also known as bearing is called as journal.



JOURNAL BEARING

Terminology of Hydrodynamic Journal Bearing:

For a hydrodynamic journal bearing the relative positions of journal and bearing with respect to various operating conditions are shown in Fig.



$$e = \text{distance } OO' = \frac{D}{2} - \frac{d}{2}$$

- Let O' be the centre of bearing
- O be the centre of journal
- D be the diameter of the bearing
- d be the diameter of journal
- l be the length of journal

1. Diametral clearance: It is the difference between the diameter of bearing and journal

$$* \quad c = D - d$$

* Generally c may be assumed as $\frac{1}{1000}$ times of journal diameter (i.e. $c = 0.001d$)

2. Diametral clearance ratio: It is the ^{ratio of} clearance to the diameter of journal.

$$* \quad \text{i.e., } c_r = \frac{c}{d} \left(= \frac{0.001d}{d} = 0.001 \text{ generally} \right)$$

3. Eccentricity (e): It is the radial distance between the centre of bearing (O') and the centre of journal (O)

$$* \quad e = \text{distance } OO' = \frac{c}{2} - h_o$$

4. Minimum film thickness: It is the minimum distance between the bearing and the journal under complete lubrication conditions

$$* \quad \text{i.e., } h_o = \frac{c}{2} - e \quad \text{where } \frac{c}{2} \text{ is called as radial clearance.}$$

5. Attitude or eccentricity ratio (ϵ): It is the ratio of the Eccentricity to the radial clearance

$$* \quad \text{i.e. } \epsilon = \frac{e}{(c/2)} = \frac{2e}{c} = \frac{2}{c} \left(\frac{c}{2} - h_o \right) = 1 - \frac{2h_o}{c}$$

Design procedure for journal bearing:

1. Determine the bearing length by choosing a ratio of l/d from table (i.e from data book)
2. Check the bearing pressure, $P = W/l \cdot d$ from table (i.e from data book)
3. Assume a lubricant from table and its operating temperature which is assumed from 60°C to 90°C
4. Determine the operating value of $\frac{Z \cdot n}{P}$ for the assumed bearing temperature and check this value with corresponding values in table (i.e from data book) to determine the possibility of maintaining fluid film operation.
5. Assume a clearance ratio c/d from table
6. Determine the coefficient of friction (μ) by using the proper relation.
7. Determine the heat generated by using the proper relation.
8. Determine the heat dissipated by using the proper relation.
9. Determine the thermal Equilibrium to see that the heat dissipated becomes atleast equal to heat generated.

In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water.

2. Check the bearing temperature $\theta = 71.9^\circ\text{C}$ from table (i.e. from data book)

3. Assume a proportional film temperature θ_{film} and its operating temperature which is assumed from 60°C to 80°C

4. Determine the operating value of $\frac{F\mu}{P}$ for the assumed bearing temperature and check this value with corresponding values in table (i.e. from data book) to determine the possibility of maintaining film temperature.

5. Assume a clearance ratio c/p from table

6. Determine the coefficient of friction (μ/p) and the load relation.

7. Determine the heat generated by the load relation.

8. Determine the heat dissipated by the load relation.

9. Determine the thermal efficiency η_{th} that the heat dissipated is equal to the heat generated.

Design a journal bearing of centrifugal pump by following data.

Speed 900rpm, oil is SAE 10, for which absolute viscosity is 55°C equal to 0.011 kg/ms . Ambient temperature of oil is equal to 15.5°C . maximum bearing pressure for the pump is 1.5 N/mm^2 . Calculate also mass of the lubricating oil required for artificial cooling. if rise of temp of oil be limited to 10°C . Heat dissipation co-efficient is $1232 \text{ watt/m}^2/^{\circ}\text{C}$.

Sol

Given data:

load acting on journal (W) = 20000 N

Speed of journal (n) = 900 rpm
oil = SAE 10

Absolute dynamic viscosity of oil (η) = 0.011 kg/ms

Ambient Atmospheric Surrounding (t_a) = 15.5°C

Allowable bearing [p] = 1.5 N/mm^2

Heat dissipation co-efficient (C) = $1232 \text{ watt/m}^2/^{\circ}\text{C}$

difference b/w inlet & outlet temp of oil (Δt) = 10°C

to find:

Design of journal bearing of centrifugal pump.

Step-1: to determine
length of journal $(l) = ?$
diameter of journal $(d) = ?$

take $\frac{l}{d}$ ratio from table 19.5

for Centrifugal pump

$$\frac{l}{d} = 1-20$$

Assume $\frac{l}{d} = 1.6$

$$l = 1.6 \times d$$

Assume $d = 100 \text{ mm}$

$$l = 1.6 \times 100$$

$$l = 160 \text{ mm}$$

Step-2:

Check the developed pressure from Eqn 19.9

$$p = \frac{W}{l \cdot d} \leq (p)$$

$$p = \frac{20000}{160 \times 100} = 1.25 \text{ N/mm}^2 \leq (p)$$

Step-3: To find bearing modulus

$$K = \frac{1}{3} \left(\frac{Zn}{p} \right)$$

take $\left(\frac{Zn}{p} \right)$ from the table.

$$K = \frac{1}{3} \times 28.45$$

$$K = 9.48$$

It operates under hydrodynamic condition

find $\frac{c}{d}$ from table 19.5 to Σ

$$\frac{c}{d} = 0.0013.$$

Step-4:

Based on units consider equation 19.5

$$\mu = \left[\frac{33.25}{10^8} \times \frac{zn}{p} \times \frac{d}{c} \right] + K$$

take $z = 0.01 \text{ kg/ms}$

$$n = 900$$

$$p = 1.25 \text{ N/mm}^2$$

$$K = 0.002$$

(from graph 19.2 based on $\frac{1}{f}$ ratio)

$$\frac{1}{f} = \frac{1}{0.0013}$$

$$\mu = \frac{33.25}{10^8} \times 12.24 \times \left(\frac{1}{0.0013} \right) + 0.002.$$

$$= 0.0053.$$

Step-5: Heat generated base on units 19.10

$$H_g = \mu \cdot \omega \cdot V$$

$V =$ sliding velocity (ω) rubbing velocity

$$\dot{V} = \frac{\pi d n}{60 \times 10^3}$$

$$= \frac{\pi \times 100 \times 900}{60 \times 10^3}$$

$$= 4.71 \text{ m/s}$$

$$H_g = 0.0053 \times 20,000 \times 4.71$$

$$= 499.26 \text{ W}$$

Step-6: Heat dissipated.

$$H_d = CA(t_b - t_a)$$

$$C = 1232 \text{ A/W}^2/\text{C}$$

$$A = l \times d$$

$$= \frac{160 \times 100}{106} = 0.016 \text{ m}^2$$

$$t_b - t_a = \frac{1}{2} (t_b + t_a)$$

$$= \frac{1}{2} (55 - 15.5)$$

$$= 19.75$$

$$H_d = 1232 \times 0.016 \times 19.75$$

$$= 389.312 \text{ W}$$

Compare H_g & H_d

⇒ the amount of artificial cooling required = $H_g - H_d$

$$= 499.26 - 389.312$$

$$= 109.948 \text{ W.}$$

⇒ Mass of oil for heat remove for artificial cooling

$$M = \frac{H_g}{c_{p \text{ oil}}} = \frac{109.94}{1900 \times 10} = 0.0057 \text{ kg/s}$$

3
The load on general bearings is 150kN due to turbine shaft of 300 dia running at 1800rpm. determine the following.

- (a) length of the bearing to the allowable bearing pressure is 1.6 N/mm^2 and
(b) Amount of heat to be removed by the lubricant per minute if the bearing temperature is 60° and viscosity of the oil at 60°C is 0.02 kg/ms and bearing clearance is 0.25 mm ?

Sol Given data.

load acting on journal (W) = 150N

diameter of journal (D) = 300mm

Speed of journal (n) = 1800 rpm.

diametral clearance (c) = 0.25mm

Absolute (or) dynamic viscosity of oil $Z = 0.02 \text{ kg/m-s}$

bearing pressure (P_b) = 1.6 N/mm^2

temperature of bearing surface (t_b) = 60°C .

to find:

length of the bearing & Amount of heat to be removed.

(i) Step-1: length of bearing

$$A = l \times d$$

$$d = 300 \text{ mm}$$

$$l = ?$$

$$A = 300 l$$

Developed bearing pressure $p = \frac{W}{l \cdot d} \leq (p)_{\text{from}} \rightarrow (19.9)$

$$p = \frac{W}{l \cdot d}$$

Here $p = 1.6 \text{ N/mm}^2$

$$W = 150 \times 10^3$$

$$d = 300 \text{ mm}$$

$$1.6 = \frac{150 \times 10^3}{l \times 300} = 8$$

$$l = 312.5 \text{ mm}$$

Step-2: Amount of heat to be removed.

Heat generated $H_g = \mu W V$ watts.

μ = coefficient of friction b/w journal & bearing.

$$\mu = \left[\frac{33.25}{10^8} \times \frac{Z_n}{p} \times \frac{d}{c} \right] + k$$

$$= \left[\frac{33.25}{10^8} \times \frac{0.02 \times 1800}{1.6} \times \frac{300}{0.25} \right] + 0.002$$

Here $k = 0.002$ [from graph 19.2 based on $\frac{l}{d}$ ratio]

$$\mu = 0.016$$

$$W = 150 \times 10^3$$

$V =$ sliding velocity (or) rubbing velocity

$$= \frac{\pi d n}{60} \text{ m/s}$$

$$d = 300 \text{ mm}$$

$$n = 1800 \text{ rpm}$$

$$= \frac{\pi \times 300 \times 1800}{60 \times 100}$$

$$= 28.27 \text{ m/s}$$

$$\therefore H_g = \mu W V$$

$$= 0.016 \times 150 \times 10^3 \times 28.27$$

$$H_g = 67848 \text{ W}$$

10) A full journal bearing of 50mm dia & 100mm long. has a bearing pressure of 1.4 N/mm^2 . the speed of journal is 900rpm & the ratio of journal dia to the dia clearance is 1000 the bearing is lubricated with oil where absolute viscosity at the operating temperature of 75° Celsius may taken as 0.01 kg/m sec the room temperature is 35°C . Find the amount of artificial cooling required.

→ the mass of the lubricated oil required if
b/w the outlet & inlet temperature is the oil is
taken specific heat of the oil has $1850 \text{ J/kg/}^\circ\text{C}$.

Q.2) Given data:

Diameter of journal (d) = 50 mm

length of journal (l) = 100 mm

bearing pressure (P_b) = 1.4 N/mm^2

ratio of journal dia to the dia clearance (d/c) = 1000 .

operating temperature $t_o = 75^\circ$

Atmospheric temperature $t_a = 35^\circ$.

Absolute (or) dynamic viscosity of oil (η) = $0.011 \text{ kg/m/}^\circ\text{C}$

Speed of journal (n) = 900 rpm .

Step-1:

Heat generated during operation

H_g : watts

μ = coefficient of friction b/w journal & bearing.

$$= \left[\frac{33.25}{10^8} \times \frac{\eta n}{P} \times \frac{d}{c} \right] + k. \rightarrow (19.5)$$

$k = 0.002$. [from graph 19.2 based on l/d ratio].

$$= \left[\frac{33.25}{10^8} \times \frac{0.011 \times 900}{1.4} \times 1000 \right] + 0.002.$$

$$= 4.35 \times 10^{-3}$$

Developed bearing pressure $\rightarrow (19.9)$

$$p = \frac{W}{l \times d} \quad \text{then.}$$

$$W = p \times l \times d$$

$$= 1.4 \times 50 \times 100$$

$$= 700$$

$$V = \frac{\pi d n}{60} \text{ m/s}$$

$$d = 50 \text{ mm}, \quad n = 900 \text{ rpm.}$$

$$= \frac{\pi \times 50 \times 900}{60 \times 1000} \text{ m/s}$$

$$= 2.356$$

$$H_g = 4.35 \times 10^{-3} \times 700 \times 2.356$$

$$= 71.74 \text{ W (or) J/s}$$

Step-2: Heat dissipated to the bearing

$$H_d = CA (t_b - t_a) \rightarrow (19.12)$$

$$C = 150 \text{ W/m}^2/\text{C} \quad [\because \text{consider } C \text{ value from (19.12)}]$$

$$A = l \times d$$

$$= 50 \times 100$$

$$= 5000 \text{ mm}^2 = 0.005 \text{ m}^2$$

$$(t_b - t_a) = \frac{1}{2} (t_b - t_a)$$

$$= \frac{1}{2} (75 - 25)$$

$$= \frac{1}{2} (40)$$

$$= 20$$

$$H_d = 150 \times 0.005 (20)$$

$$= 15 \text{ W}$$

step-3: Mass of oil for heat removal

$$M = \frac{H_d}{c_{p \text{ oil}}} \rightarrow (19.13)$$

$$= \frac{71.74}{1850 \times 10}$$

$$= 3.87 \times 10^{-3}$$

$$= 0.00387 \text{ kg/s}$$

pb) A 150mm diameter shaft supporting a load of 10kN has a speed of 1500 rpm. The shaft runs in a bearing whose length is 1.5 times the shaft diameter. If the diameter clearance of the bearing is 0.15 mm and the absolute viscosity of the oil at the operating temperature is 0.01 kg/m-s find the power registered in friction (heat generated).

Given data

6

diameter of journal (d) = 150 mm

load acting on the journal (w) = 10×10^3 N.

Speed of journal (n) = 1500 rpm.

diameter clearance (c) = 0.15 mm

Absolute (3) dynamic viscosity of oil (η) = 0.01 kg/m.s

length of journal $l = 1.5(d)$.

step-1: length of journal
 $l = 1.5(d)$
 $= 1.5 \times 150$
 $= 225$ mm.

step-2: Developed bearing pressure

$$p = \frac{W}{l \cdot d}$$
$$= \frac{10 \times 10^3}{225 \times 150}$$
$$= 0.29 \text{ N/mm}^2.$$

$\therefore$ $W = 10 \times 10^3$
 $l = 225$ mm, $d = 150$ mm

step-3: Heat generated during operation

$$H_g = \mu W V$$

μ : coefficient of friction b/w journal & bearing

$$\mu = \left[\frac{33.25}{10^8} \times \frac{2n}{p} \times \frac{d}{c} \right] + K$$

$$K = 0.002 \quad \left[\text{based from graph (9.2) based on } \frac{1}{f} \text{ ratio} \right]$$

$$= \left[\frac{33.25}{10^8} \times \frac{0.011 \times 1500}{0.29} \times \frac{150}{0.15} \right] + 0.002$$

$$= 0.02$$

$$W = 10 \times 10^3 \text{ N.}$$

$$v = \frac{\pi d n}{60} \text{ m/s}$$

$$d = 150 \text{ mm}, \quad n = 1500$$

$$V = \frac{\pi \times 150 \times 1500}{60 \times 1000}$$

$$= 11.78$$

$$H_f = \mu W V$$

$$= 0.02 \times 10 \times 10^3 \times 11.78$$

$$= 2414.9 \text{ W.}$$

$$\text{Power wasted in friction} = 2414.9 \text{ W.}$$

80mm long general bearing supports a load of 2800N on a 50mm dia shaft the bearing has a radial clearance of 0.05mm and the viscosity of the oil is 0.021 kg/m.s at the operating temperature if the bearing is capable of dissipating 80 Joules/s . Determine the maximum safe speed.

30) Given data.

$l = \text{length of journal} = 80\text{mm}$

load acting on the journal (W) = 2800N

diameter of journal (d) = 50mm .

Absolute (or) dynamic viscosity of oil (η) = 0.021 kg/m.s

Heat dissipating (H_d) = 80 J/s

radial clearance $c/d = 0.05 = 0.1$

step-1: Heat generated during operation

Developing bearing pressure

$$p = \frac{W}{P \cdot l}$$

$$\left[\begin{array}{l} W = 2800 \\ l = 80\text{mm}, d = 50\text{mm} \end{array} \right]$$

$$= \frac{2800}{80 \times 50} = 0.7\text{ N/mm}^2$$

μ = coefficient of friction b/w journal & bearing

$$\mu = \left[\frac{33.25}{10^8} \times \frac{Zn}{P} \times \frac{d}{c} \right] + k$$

$k = 0.02$ [from graph 19.2 based on l/d ratio]

$$\mu = \left[\frac{33.25}{10^8} \times \frac{0.02 \times n}{0.7} \times \frac{50}{0.1} \right] + 0.002$$
$$= 4.9875 \times 10^{-6} \times n + 0.002$$

$$W = 2800 \text{ N}$$

$$V = \frac{\pi d n}{60} \text{ m/s}$$

$$= \frac{\pi \times 50 \times n}{60 \times 1000}$$

$$= 2.617n \times 10^{-3}$$

$$H_g = \mu W V$$

$$= [4.9875 \times 10^{-6} \times n + 0.002] \times (2800) \times (2.617 \times 10^{-3} n)$$

$$= 3.65 \times 10^{-5} n^2 + 0.014n$$

step 2: Heat dissipated by the bearing = Heat generated

$$H_g = H_d$$

$$3.65 \times 10^{-5} n^2 + 0.014n = 80$$

$$= 3.65 \times 10^{-5} n^2 + 0.014n - 80 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-0.014 \pm \sqrt{(0.014)^2 - 4(3.65 \times 10^{-5})(-80)}}{2(3.65 \times 10^{-5})}$$

$$n = 1287.67 \text{ rpm}$$

Formula's of Deep groove ball bearing:

⇒ Dynamic capacity (or) dynamic load rating:

$$C = \left(\frac{L}{L_0} \right)^{1/k} \cdot p \quad \text{Where } k=3 \text{ for ball bearings}$$

$$L_0 = 1 \text{ Mr}$$

⇒ Equivalent load:

$$P = (VXFr + YFa)S$$

$V=1.0$ for inner ring rotation and outer ring stationary

$F_r, F_a \rightarrow$ Radial and axial loads

$X, Y \rightarrow$ Radial and axial load factors.

$S \rightarrow$ Service factor.

⇒ Life of bearings in million revolutions:

$$L = (C/P)^k \text{ Mr.}$$

⇒ life of bearings in hours

$$L_h = \frac{L \times 10^6}{60 \times n} \text{ hours.}$$

⇒ Cubic mean load for periodically variable loading

$$P_m = \left[\frac{P_1^3 n_1 + P_2^3 n_2 + \dots + P_r^3 n_r}{n_1 + n_2 + \dots + n_r} \right]^{1/3} \text{ for revolutions varying}$$

$$= \left[\frac{P_1^3 t_1 + P_2^3 t_2 + \dots + P_r^3 t_r}{t_1 + t_2 + \dots + t_r} \right]^{1/3} \text{ for time varying.}$$

⇒ mean load for linearly variable loading

$$P_{\text{mean}} = \frac{P_{\text{min}} + 2P_{\text{max}}}{3}$$

⇒ probability of survival:

$$\frac{L}{L'_{10}} = \left[\frac{\ln(1/p)}{\ln(1/p_{10})} \right]^{1/b}$$

Where L = required life of bearings in mr

L'_{10} = calculated life of selected bearings, for the given load, for 90% survival.

p = probability of survival

$$\ln(1/p_{10}) = \ln(1/0.9) = 0.1053, \quad b = 1.17 \text{ for a median life} \\ = 5L_{10}$$

select a single row deep ball bearing for a radial load of 4000N & axial load of 5000N operating with a speed of 160 rpm. for an average life of 5 years at 10 hours per day. Assume uniform & steady load?

Given data:-

Radial load (F_r) = 4000N

axial load (F_a) = 5000N

speed of the bearings (n) = 1600 rpm

Assume d = 70mm

Step-1: Dynamic capacity (a) dynamic load rating

$$C = \left(\frac{L}{L_0} \right)^{1/k} \cdot P \rightarrow (20.1)$$

L_h = life of bearings in hours

$$L_h = 5 \times 300 \times 10 \\ = 15000 \text{ hours}$$

Step-2: Life of bearings in hours

$$L_h = \frac{L \times 10^6}{60 \times n} \rightarrow (20.4)$$

$$L_h = 15000, \quad n = 1600 \quad \text{then}$$

$$L = \frac{L_h \times 60 \times n}{10^6}$$

$$= \frac{15000 \times 60 \times 1600}{10^6}$$

$$L = 1440 \times 10^6$$

step-3 : Equivalent load

$$P = (VX F_r + Y F_a) S$$

$V = 1.0$ for inner ring rotation and outer ring stationary

$S =$ service factor (table 20.3)

$$V = 1, S = 1$$

Assume :

$$\frac{F_a}{C_0} = 0.025$$

$$e = 0.22$$

[from table 20.5]

$$\frac{F_a}{F_r} = \frac{5000}{4000} = 1.25$$

$$\frac{F_a}{F_r} > e \quad \text{then}$$

$$X = 0.56, Y = 2$$

$$P = (0.56 (4000) + 2 (5000)) (1)$$

$$= 12240$$

from step-1 $\rightarrow C = \left(\frac{1440 \times 10^6}{10^6} \right)^{1/3} \times 12240$

$$C = 138.219 \times 10^3 \text{ N}$$

Based on 'C' value we have to take bearing number & static number from the data book from the table (20.17).

$$\text{Bearing number} = 6321$$

$$C_0 = 140240, \text{ then}$$

$$\Rightarrow f_a / C_0 = \frac{5000}{140240} = 0.0356$$

$$\frac{f_a}{C_0} = 0.04 \quad [\text{from data book table 20.5}]$$

$$e = 0.24$$

$$\frac{f_a}{f_r} = 1.25 \Rightarrow \left[\frac{5000}{4000} = 1.25 \right]$$

$$\frac{f_a}{f_r} > e \Rightarrow [\text{If } \frac{f_a}{f_r} > e \text{ then consider } x \text{ \& } y \text{ values from table 20.5}]$$

$$x = 0.56$$

$$y = 2 -$$

$$y = 2 - \frac{(2 - 1.8)}{(0.04 - 0.025)} \times (0.04 - 0.0356)$$

$$= 1.94$$

Equivalent load

$$P = (VX F_r + Y F_a) S$$

$$= (0.56 \times 4000 + 1.94 \times 5000) (1)$$

$$= 11940$$

from step - 1

$$C = \left(\frac{L}{L_0} \right)^{1/k} (P)$$

$$= \left(\frac{1440 \times 10^6}{10^6} \right)^{1/3} \times 11940$$

$$C = 134.831 \times 10^3 \text{ N}$$

$$= 134831 \text{ N}$$

Based on 'C' value we have to take bearing number from the data book from the table (20.17)

Bearing Num = 6320

select a single row deep groove ball bearing for a radial load of 7000 N & a thrust load of 2100 N the desired life of bearing is 160 Mr of revolution at 300 rpm. Assume static load of 72000 N. And the diameter of the shaft is 75 mm?

Given data.

Radial load $F_r = 7000 \text{ N}$

Axial load $F_a = 2100 \text{ N}$

Speed of the bearings $(n) = 300 \text{ rpm}$

Life of bearing in million revolutions $L = 160 \text{ Mr}$

Static load rating $C_0 = 72000 \text{ N}$

diameter of the shaft $d = 75 \text{ mm}$.

Step-1: Dynamic Capacity (or) dynamic load rating

$$C = \left[\frac{L}{L_{10}} \right]^{1/k} \cdot P \rightarrow (20.1)$$

$$L = 160 \text{ Mr} \Rightarrow 160 \times 10^6$$

$$L_{10} = 1 \text{ Mr} = 10^6 \quad [\text{from data book } (20.1)]$$

$$k = 3 \quad [\text{taken from } (20.1)]$$

$$P = ?$$

Step-II: Equivalent load

$$P = [VXFr + YFa]S$$

$V=1.0$ for inner ring rotation and outer ring stationary

Fr, Fa = Radial and axial loads

X, Y = Radial and axial load factors

S = Service factor.

Assume $V=1$
 $S=1$

$$\frac{Fa}{Co} = \frac{\text{Axial load}}{\text{static load rating}}$$

$$= \frac{2100}{72000} = 0.0291$$

$e = 0.22 \rightarrow$ [from data book. table num (20.5)]

$$\frac{Fa}{Fr} = \frac{2100}{7000} = 0.3$$

$$\frac{Fa}{Fr} > e$$

$X=0.56, Y=2$ [from data book table number (20.5)]

Substitute all values in 'P'.

$$P = [1(0.56)(7000) + 2(2100)] \times (1)$$
$$= 8120$$

Step-III: Life of bearings in hours

$$L_h = \frac{L \times 10^6}{60 \times n}$$

$$= \frac{160 \times 10^6}{60 \times 300}$$

$$L_h = 8888.88 \text{ hrs.}$$

"Substitute $p = 8120$ value in step-I"

$$C = \left[\frac{160 \times 10^6}{10^6} \right]^{1/3} \cdot 8120$$
$$= 44082.14 \text{ N}$$

bearing number = 62124

$$C_0 = 38440$$

Based on 'C' value we taken the bearing number & static number from data table from the (20.15) book

$$\Rightarrow \frac{f_a}{C_0} = \frac{2100}{38440}$$

$$= 0.058$$

$$\text{Assume } \frac{f_a}{C_0} = 0.07$$

$$e = 0.27$$

$$\frac{f_a}{f_r} = 0.3$$

$$\frac{f_a}{f_r} > e$$

$$x = 0.56 \quad \left[\text{taken from data book from table (20.5)} \right]$$

$$y = \text{highest } y \text{ value} - \frac{(\text{difference b/w } y \text{ value})}{(\text{difference b/w } f_a/c_o)} \times \left(\text{data book value} - \text{calculated value} \right)$$

$$y = 2 - \frac{(2 - 1.6)}{(0.07 - 0.0291)} \times [0.07 - 0.058]$$

$$= 0.4694$$

use the interpolation we get
y value.

$$p = (V \times F_r + Y \times F_a) S$$

$$= [0.56 \times 7000 + 0.4694 \times 2100]$$

$$= 4905.74$$

$$C = \left(\frac{L}{L_{10}} \right)^{1/k} \cdot p$$

$$= \left(\frac{160 \times 10^6}{10^6} \right)^{1/3} \cdot 4905.74$$

$$C = 26632.45 \text{ N}$$

Based on 'i' value we have to take bearing number from the data book from the table (20.15)

Bearing number is 6210

13

Single groove ball bearing has a dynamic load capacity of 40,500N and operates at the following work cycle Radial load of 5000N at 500rpm for 25% of the time, Radial load of 10,000N at 700rpm for 50% of the time, Radial load of 7000N at 400rpm for 25% of the time. Calculate the expected life of bearing in hrs?

Given data:

Dynamic load capacity $(C) = 40,500\text{N}$.

S.No	load	time	Speed	element time of revolution
1	5000N	0.25	500	125
2	10,000N	0.5	700	350
3	7000N	0.25	400	100
				$n = 575$

$n \rightarrow$ speed of the bearings.

Step-1: Cubic mean load for periodically variable loadings

$$P_m = \left[\frac{P_1^3 n_1 + P_2^3 n_2 + P_3^3 n_3}{n_1 + n_2 + n_3} \right]^{1/3} \quad \text{from data book (20.5)}$$

Here $P_m =$ Cubic mean load.

$$= \left[\frac{(5000)^3 (125) + (10,000)^3 (350) + (7000)^3 (100)}{125 + 350 + 100} \right]^{1/3}$$

$$= 7855.08 \text{ N}$$

$$= 8860.02 \text{ N}$$

step-II: life of bearings in million revolutions

$$L = \left(\frac{C}{P} \right)^k \quad \text{from data book (20.3)}$$

Here $C = 40500 \text{ N}$

$$P = 8860.02 \text{ N}$$

$$k = 3$$

$$\text{then } L = \left(\frac{40500}{8860.02} \right)^3$$

$$= 95.51 \text{ rev.}$$

step-III: life of bearing in hours

$$L_h = \frac{L \times 10^6}{60 \times n}$$

L : life of bearing in million revolutions = 95.51
 n : speed of the bearings = 575

$$L_h = \frac{95.51 \times 10^6}{60 \times 575}$$

$$L_h = 2768.40 \text{ hrs}$$

14
 A single row deep groove ball bearing is subjected to a radial force of 8 kN and a thrust force of 3 kN. The values of X & Y factors are 0.56 and 1.5 respectively. The shaft rotates at 1200 rpm. The diameter of the shaft is 75 mm and bearing number is 6315.

($C = 112000 \text{ N}$) is selected for this application?

- Estimate the life of this bearing with 90% reliability
- Estimate the reliability of 20,000 hrs life.

sol) Given data:

Radial load $F_r = 8 \text{ kN} = 8 \times 10^3 \text{ N}$
 Thrust load $F_a = 3 \text{ kN} = 3 \times 10^3 \text{ N}$

Radial load factor (X) = 0.56
 Thrust load factor (Y) = 1.5

Speed of the bearings (n) = 1200 rpm.

Diameter of the shaft (d) = 75 mm.

Bearing number = 6315, ($C = 112000 \text{ N}$).

Step 1: Life of bearing in million revolutions

$$L = (C/P)^k \rightarrow (20.1)$$

$$= \left(\frac{112000}{8980} \right)^3 = 1940.10 \text{ Mr.}$$

Here $p = (VXFr + \gamma Fa)S. \rightarrow$ Equivalent load

$$= (11 \cdot 60.56) (8 \times 10^3) + (1.5) (3 \times 10^3) \quad (1)$$
$$= 8980 \text{ N.}$$

step-II: Estimate the life of bearing.

life of bearing in hours

$$L_h = \frac{L \times 10^6}{60 \times n} \rightarrow (20.4)$$

$$= \frac{1940 \times 10 \times 10^6}{60 \times 1200}$$

$$= 26945.83 \text{ hrs.}$$

step-III: probability of survival.

$$\frac{L}{L_{10}} = \left[\frac{\ln(1/p)}{\ln(1/p_{10})} \right]^{1/b}$$

$$\frac{20,000}{26945.8} = \left[\frac{\ln(1/p)}{0.1053} \right]^{1/1.7}$$

$$\ln(1/p) = \left(\frac{20,000}{26945.8} \right)^{1.7} \times 0.1053$$

$$= 0.07429$$

$$1/p = e^{0.07429}$$

$$p = \frac{1}{e^{0.07429}}$$

$$p = 0.928 \Rightarrow 92.8\%$$

Single groove deep ball bearing is subjected a 30 sec. work cycle following two parts.

	Part - I	Part - II
duration (s)	10	20
radial load (kN)	45	15
axial load (kN)	12.5	6.25
Speed (rpm)	720	1440

The static and dynamic load capacities of ball bearing are 50, 68 kN respectively calculate the expected life of bearing in hrs?

Given data:

dynamic load capacities $C = 50, 68 \text{ kN}$

	part - I	part - II
duration (s)	10	20
radial load (kN) (F_r)	45	15
axial load (kN) (F_a)	12.5	6.5
Speed (rpm)	720	1440

For Part - I

$$\frac{F_a}{C_0} = \frac{12.5}{50} = 0.25$$

based on $\frac{f_a}{C_0}$ we have to take the 'e' value
data book from the table 20.5

$$e = 0.37$$

$$\frac{f_a}{f_r} = \frac{12.5}{45} = 0.27$$

$\frac{f_a}{f_r} < e$. (based on this step we have to take x, y values from data book from the table 20.5)

$$x = 1 \quad y = 0$$

Equivalent load $P_1 = (x f_r + y f_a) S \rightarrow (20.2)$
 $= (1 \times 45 + 0 \times 12.5) (1)$

$$P_1 = 45 \times 10^3 \text{ N}$$

Ex 8 part - II

$$\frac{f_a}{C_0} = \frac{6.25}{50} = 0.125$$

$$e = 0.31$$

based on $\frac{f_a}{C_0}$ we have to take the 'e' value from
data book from the table 20.5

$$\frac{f_a}{f_r} = \frac{6.25}{15} = 0.416$$

$\frac{F_a}{F_r} > e$ (based on this step we have to take x value from data book from the table 20.5)

$$x = 0.56$$

$$Y = 1.6 - \left[\left(\frac{16 - 1.4}{0.13 - 0.07} \right) \times (0.13 - 0.185) \right] = 1.583.$$

Equivalent load $P_2 = [V \times F_{r2} + Y F_{a2}] S$

$$N_2 = \frac{20}{60} \times 1440 = 480$$

$$N_1 + N_2 = 120 + 480 = 600 \rightarrow$$

$$\text{one minute} = 1200 \Rightarrow n = 1200$$

Cubic mean load for periodically variable loading

$$P_m = \left[\frac{P_1^3 N_1 + P_2^3 N_2}{N_1 + N_2} \right]^{1/3}$$

$$= \left[\frac{(450 \times 10^3)(120) + (18.293 \times 10^3)(480)}{600} \right]^{1/3}$$

$$P_e = 28488.93 \text{ N.}$$

Life of bearing in million revolution

$$L = \left(\frac{C}{P} \right)^K$$

$$= \left(\frac{68 \times 10^3}{28488.93} \right)^3$$

$$L = 13.598 \text{ mrev}$$

life of bearing in hours

$$L_h = \left[\frac{L \times 10^6}{60 \times n} \right]$$

$$= \frac{13.598 \times 10^6}{60 \times 1200}$$

$$L_h = 188.86 \text{ hrs.}$$

Engine parts :Connecting rod :

The connecting rod is the intermediate member between the piston and the crankshaft. Its primary function is to transmit the push and pull from the piston pin to the crank pin and thus convert the reciprocating motion of the piston into the rotary motion of the crank.

It consists of a long shank, a small end and a big end. The cross section of the shank may be rectangular, circular, tubular, I-section or H-section. Generally circular section is used for low speed engines while I-section is preferred for high speed engines.

It consists of an eye at the small end (so called due to its small size) for connecting the piston through piston pin, a long shank usually of I-section, and a big end, which connected to the crankshaft pin, of split type and has a separate cap. The cap is secured to the body of the rod by means of two or four bolts. In large sized connecting rods, holes are provided for lubricating purposes.

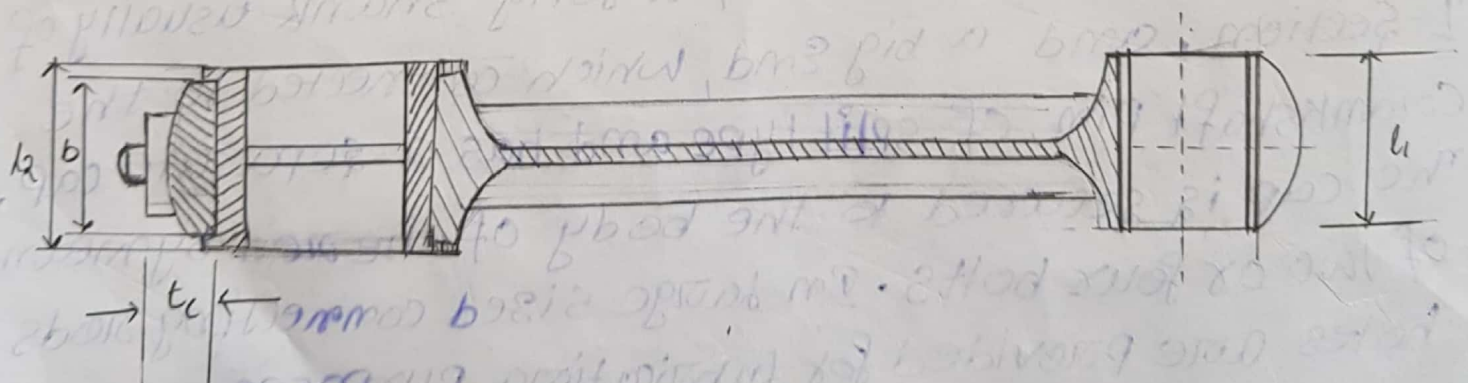
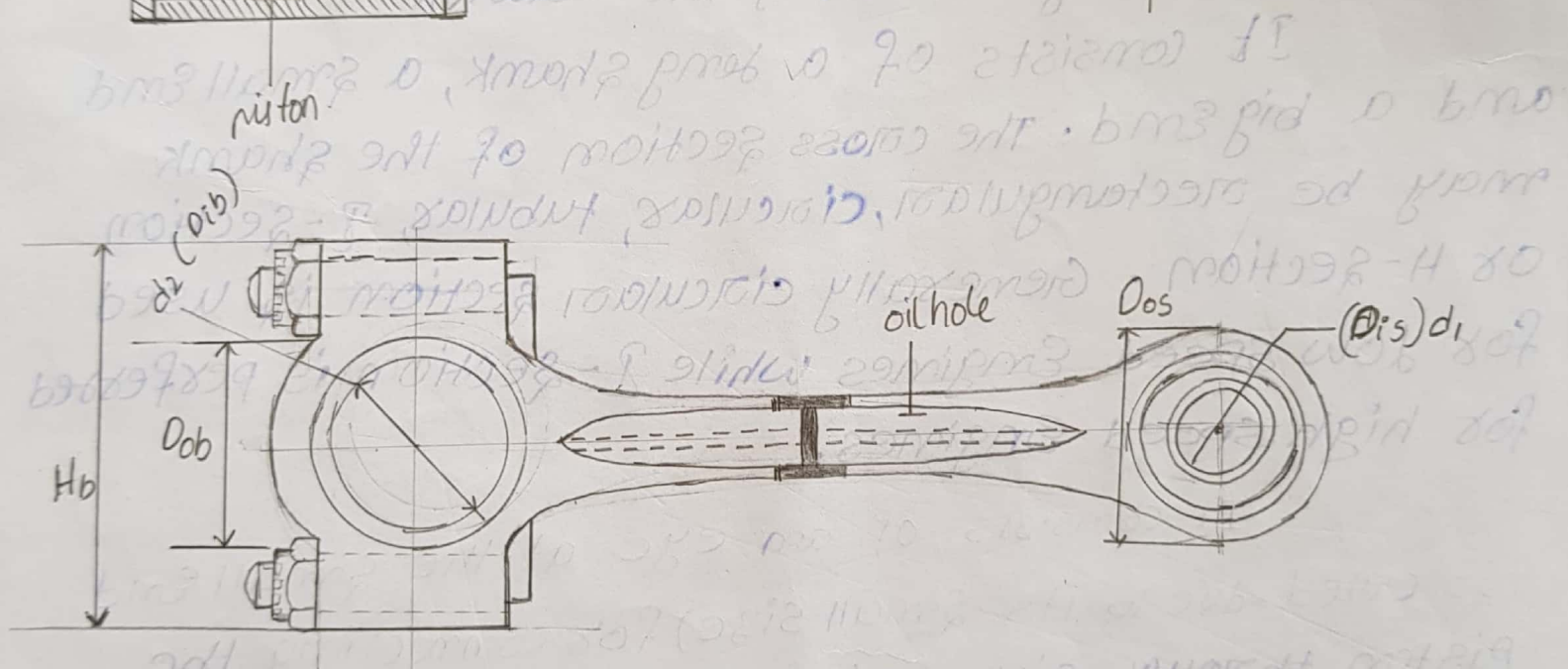
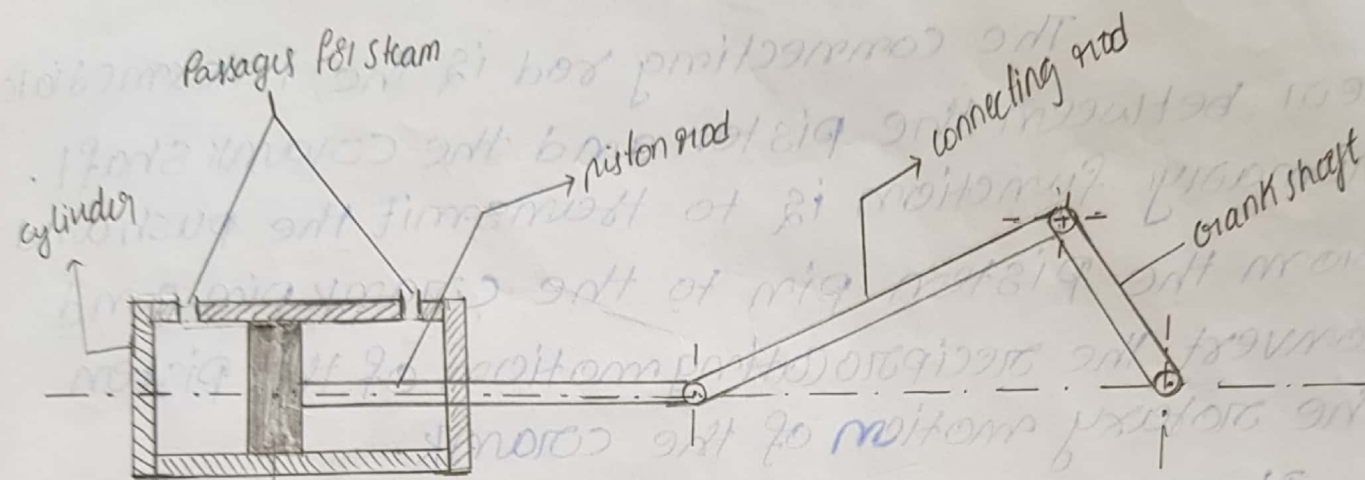


FIG 1: CONNECTING ROD OF AN I.C. ENGINE

Design procedure for connecting Rod :

(3)

For the design of connecting rod, the following steps may be observed.

1. From the statement of problem, note the pressure of steam or gas, length of connecting rod, crank radius etc. Then select suitable material usually mild steel for the connecting rod and find its design stresses. Assume the essential non-given data suitably based on the working conditions.
2. Select I-section connecting rod if possible and determine its moment of inertia about X-axis, and Y-axis.
3. Equate the steam force with buckling strength of connecting rod using Rankine's formula and determine the dimensions of connecting rod.
4. Calculate the maximum bending stress and then compare it with design stress of the connecting rod for checking.

cylinder: The function of a cylinder is to retain the working fluid and to guide the piston. The cylinders are usually made of cast iron or cast steel. Since the cylinder has to withstand high temperature due to the combustion of fuel, therefore some arrangement must be provided to cool the cylinder. The single cylinder engines (such as scooters and mopeds) are generally air cooled. They are provided with fins around the cylinder.

The multi cylinder engines (such as of cars) are provided with water jackets around the cylinders to cool it. In small engine the cylinder, water jacket and the frame are made as one piece, but for all the larger engines, these parts are manufactured separately. The cylinders are provided with cylinder liners so that in case of wear, they can be easily replaced.

A cylinder liner which doesn't have any direct contact with the engine cooling water, is known as dry liner. A cylinder liner which have its outer surface in direct contact with the engine cooling water, is known as wet liner.

select the web parameters proportionately & check their induced stresses.

6. In any case, if the induced stress is more than the allowable value, then alter the corresponding dimension suitably.

7. Usually, the following proportions are adopted for the crank-shaft parts.

Let d = Diameter of Crank pin

D = Diameter of main journal.

Then for overhung crankshaft

- (a) Diameter of main journal (D) = 1.25 to $1.5d$
- (b) Length of main journal, (L) = $1.25D$
- (c) Length of journal inside the crank, (L_1) = 1.0 to $1.25D$
- (d) length of crank pin, (l) = 1.0 to $1.25d$
- (e) Length of pin inside the crank, (l_1) = 1.0 to $1.25d$
- (f) thickness of web, (b) = 0.7 to $1.0d$
- (g) width of web nearer to crank-pin, (a) = $1.5d$
- (h) width of web nearer to journal, (b) = $1.5D$

For centre crankshaft;

(a) Diameter of journal, $(\phi) = d$

(b) Thickness of web, $(t) = 0.7d$

(c) width of web, $(w) = 1.5d$

CRANK SHAFT :

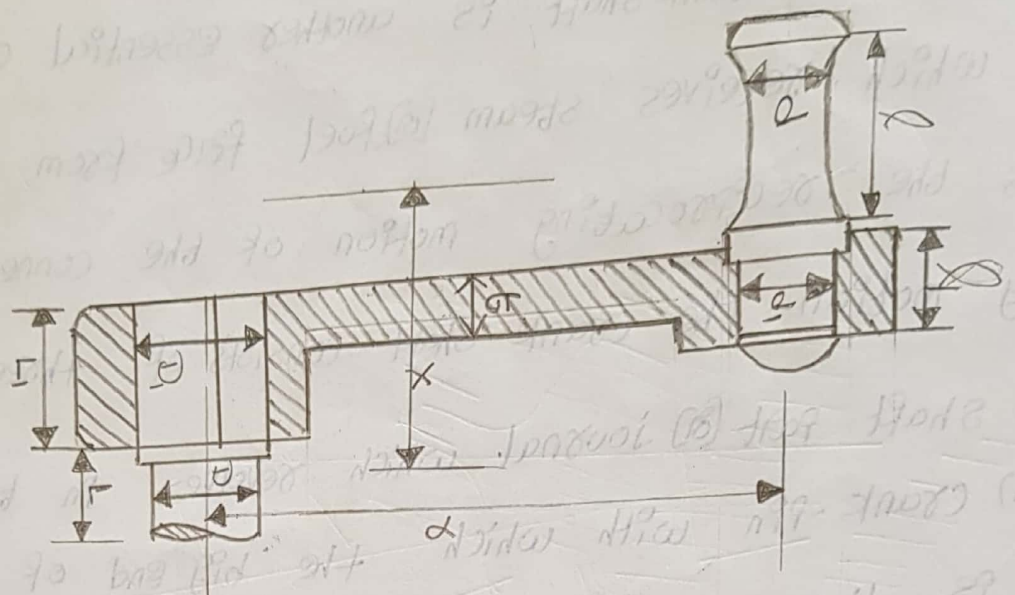
crank shaft is another essential component of the engine which receives steam [or] fuel force from connecting rod & converts the reciprocating motion of the connecting rod into a rotary motion. The crank shaft consists of three main parts namely

- (i) shaft part [or] journal which revolves in the main bearing,
- (ii) crank-pin with which the big end of the connecting rod is linked and
- (iii) Crank-web (also known as cheek) which unites the crankpin with the journal.

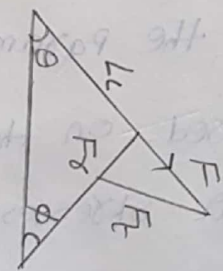
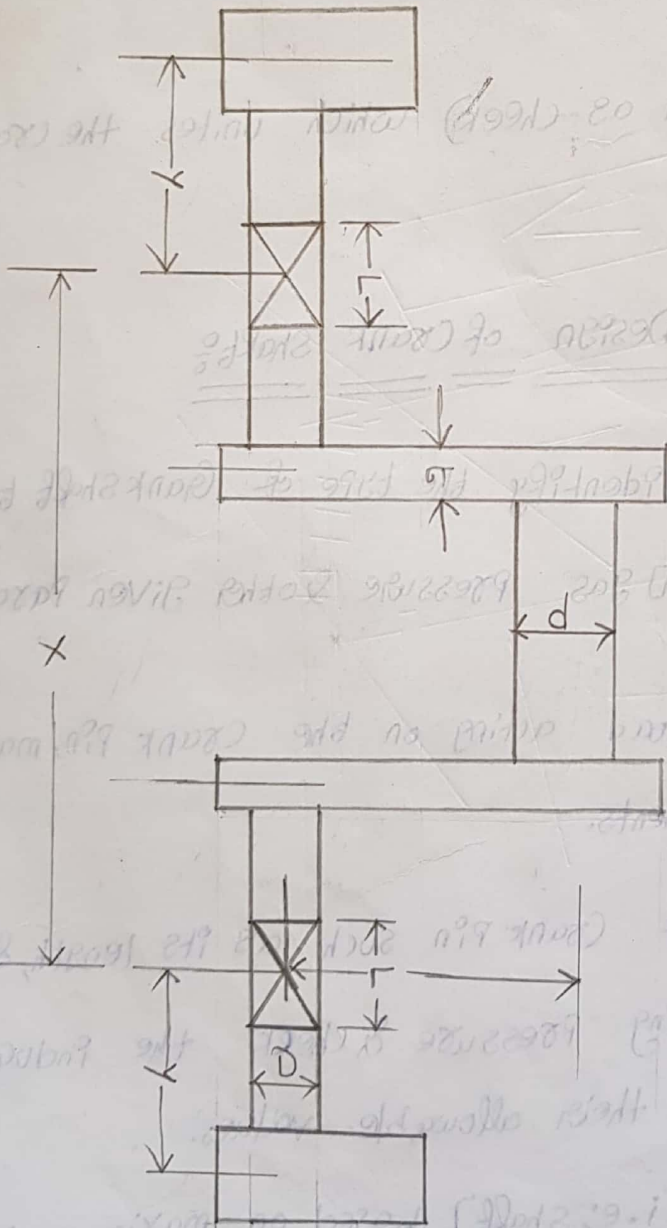
Steps Involved in the Design of Crank Shaft :

1. From the given problem, identify the type of Crankshaft to be designed, material, steam [or] gas pressure & other given parameters.
2. Determine the maximum load acting on the crank pin, maximum torque and bending moments.
3. Find out the parameters of Crank pin such as its length, & diameter etc. based on the bearing pressure & check the induced bending & shear stresses with their allowable values.
4. Design the main journal [i.e., shaft] based on maximum torque & bending moment conditions & check the bearing pressure.

over hung crank shaft



center crankshaft



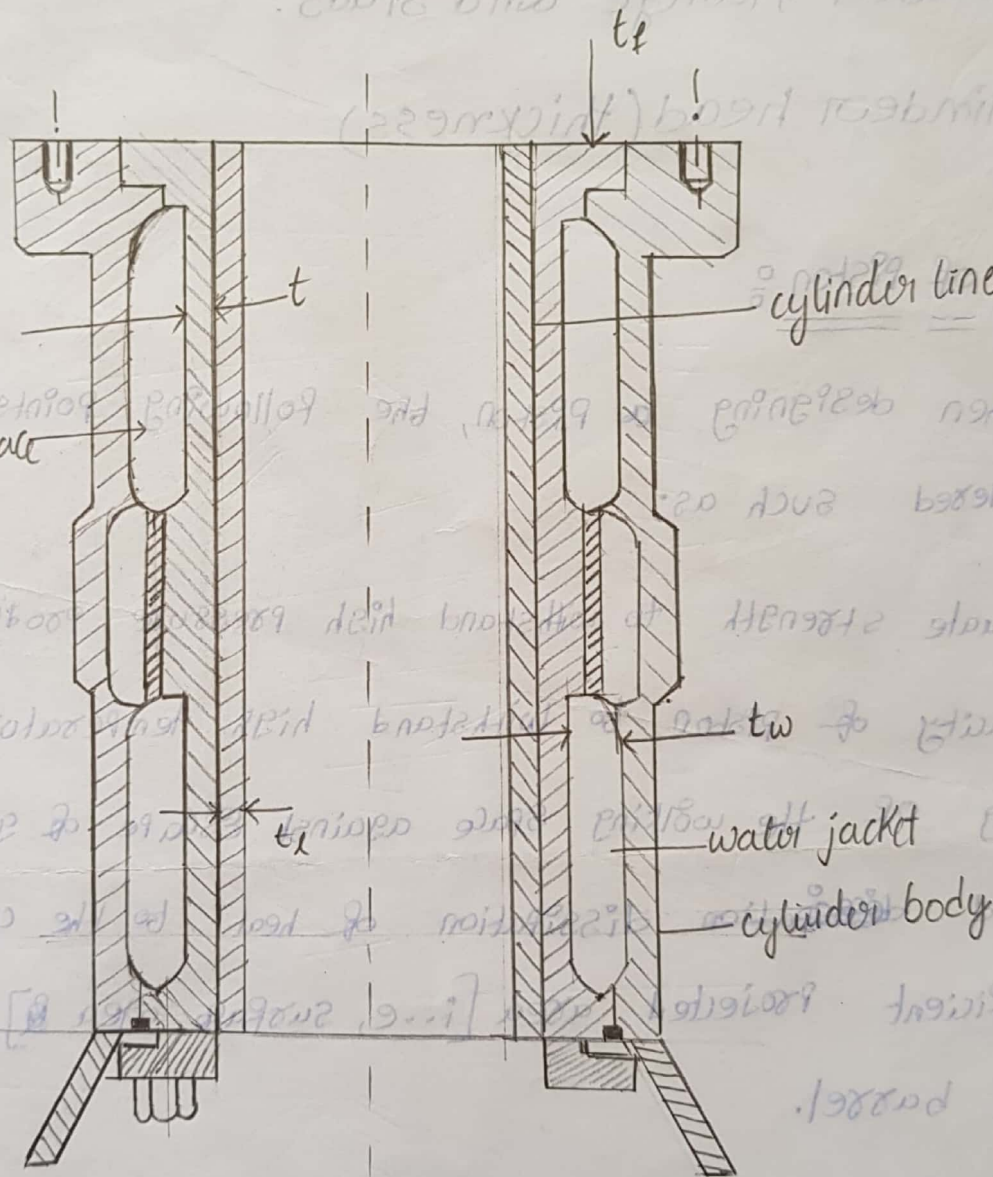


FIG: I.C. ENGINE - CYLINDER

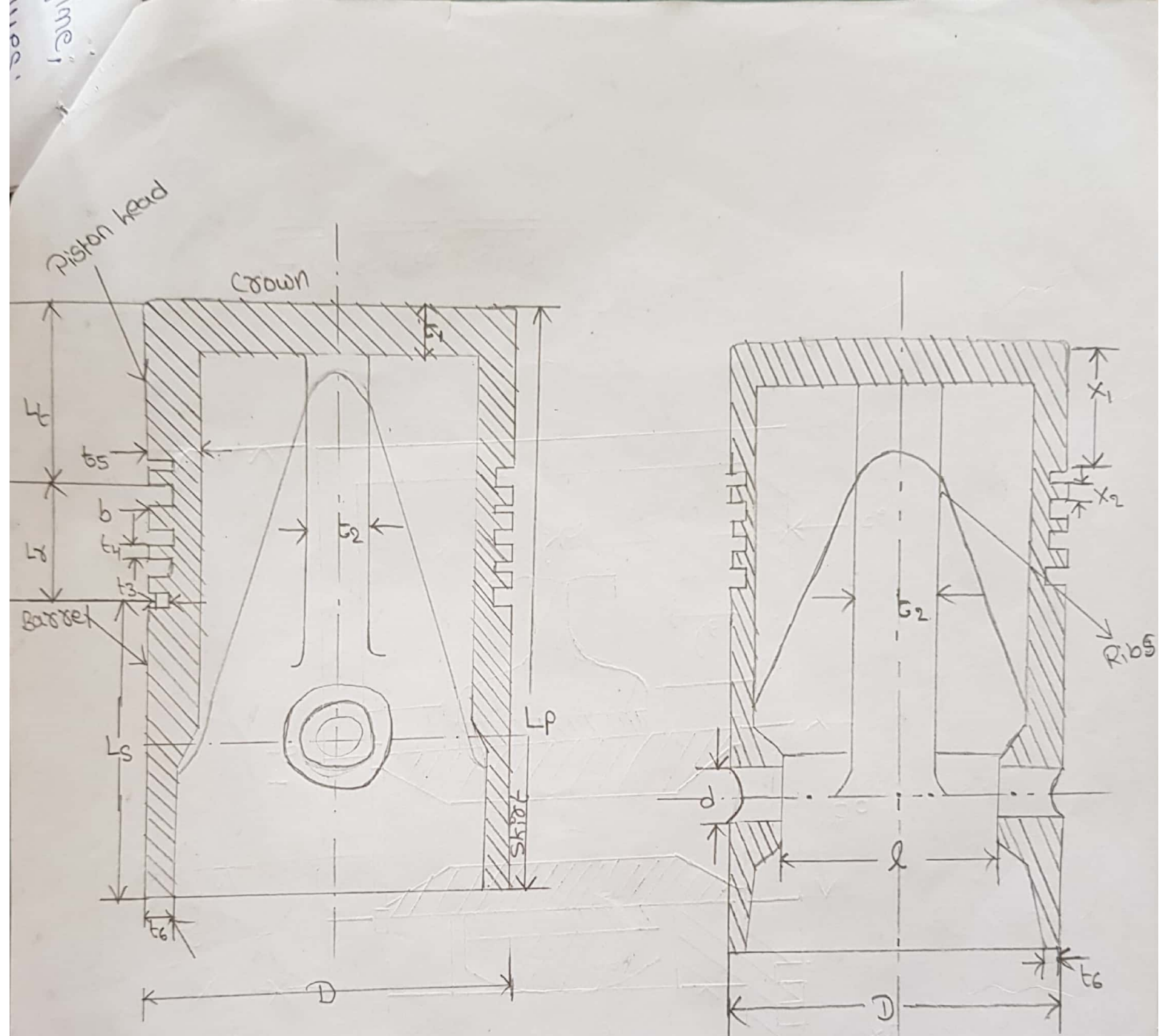
In designing a cylinder for an I.C Engine, it is required to determine the following values:

1. Thickness of the cylinder wall
2. Bore and length of the cylinder
3. Cylinder flange and studs.
4. Cylinder head (thickness)

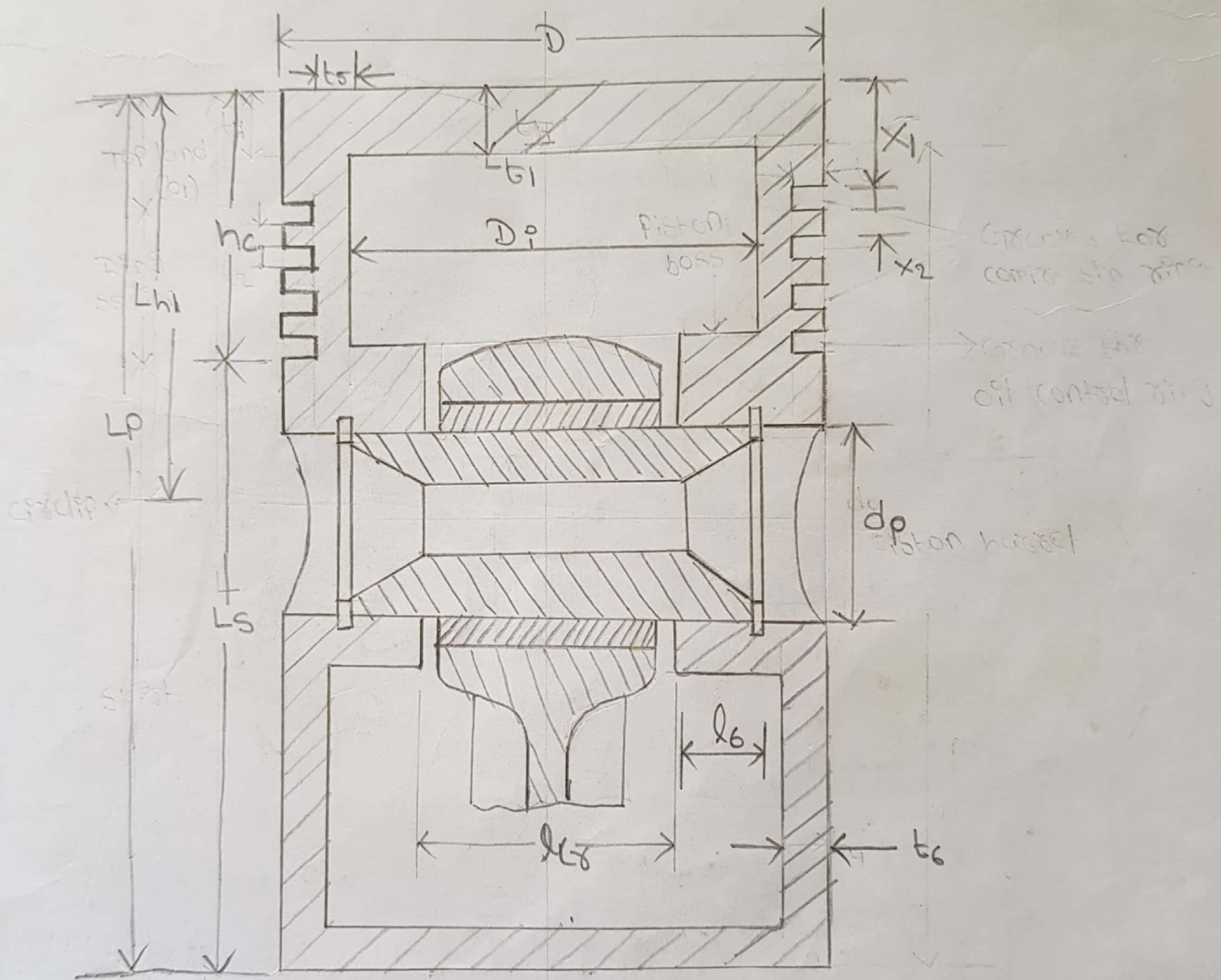
Design of Piston:

When designing a piston, the following points must be considered such as:

1. Adequate strength to withstand high pressure produced by the gas.
2. Capacity of piston to withstand high temperature.
3. Sealing of the working space against escape of gases.
4. Good ~~dissipation~~ dissipation of heat to the cylinder wall.
5. Sufficient projected area [i.e., surface area] & rigidity of the barrel.
6. Minimum loss of power due to friction.
7. Minimum weight, to reduce inertia force & unbalanced effects.
8. Sufficient length to have better guidance & so on



Piston



I.C. engine piston

UNIT-II

ENGINE PARTS

- * PISTON
- * CONNECTING ROD
- * CYLINDER
- * CRANK SHAFT

UNIT-2

Engine parts

* Design a cast iron piston for a single acting 4 stroke engine for the following data.

Bore = 100mm, stroke = 125mm, Max gas pressure = 5 N/mm².
Indicated mean effective pressure is 0.75 N/mm² and Mechanical efficiency is 80%. Fuel consumption 0.15 kg/break power/hr. Higher - calorific value is 42 x 10³ KJ/kg. Speed 2000 rpm. Any other data required for the design may be assumed.

Sol: We need to design a cast iron piston.

Given data:

Diameter of piston $[D] = 100\text{mm}$

Length of stroke $[L] = 125\text{mm}$

Maximum gas pressure $[Max P] = 5\text{ N/mm}^2$

Mean effective pressure = 0.75 N/mm^2

Mechanical Efficiency $[\eta_m] = 80\% = 0.8$

calorific value $[C_v] = 42 \times 10^3\text{ KJ/kg}$

speed of crank $[N] = 2000\text{ rpm}$

Mass of fuel consumed $[m] = 0.15\text{ kg/BP/hr}$

⇒ As, fuel consumed is in hourly, we need convert that into seconds,

$$\text{So, } \frac{0.15}{3600} = 4.16 \times 10^{-5}\text{ kg/BP/sec}$$

step I: Thickness of piston head based on the strength of piston material.

$$t_1 = \sqrt{\frac{3 P_m D^2}{16 \sigma_{tp}}} \text{ mm} \rightarrow \text{(eq. 15.13) (Data Book)}$$

$$= \sqrt{\frac{3 \times 5 \times (100)^2}{16 \times 40}}$$

$$= 15.30 \text{ mm}$$

$$\approx 16 \text{ mm (Approximately)}$$

step II: Thickness of piston head based on heat dissipation

$$t_1 = \frac{1000 H}{12.56 K (T_c - T_e)} \rightarrow \text{(eq. 15.14) (Data Book)}$$

$$\Rightarrow H = 0.05 \text{ m} \times C_v \times P_{\text{I}} \text{ kW}$$

$$\Rightarrow P_{\text{I}} = \frac{P_{\text{LAN}}}{60 \times 10^6} = \frac{0.7 \times \frac{\pi}{4} D^2 \times 125 \times 1000}{60 \times 10^6}$$

$$= 11.450 \text{ kW}$$

$$\Rightarrow P_B = P_{\text{I}} \times \eta_m = 11.45 \times 0.8 = 9.16 \text{ kW}$$

$$\Rightarrow H = 0.05 \times 9.16 \times 10^{-5} \times 42 \times 10^3 = 9.16$$

$$= 0.80 \text{ kW}$$

$$\rightarrow \text{Now, } t_1 = \frac{1000 H}{12.56 K (T_c - T_e)}$$

$$= \frac{1000 \times 0.80}{12.56 \times 46.6 \times 10^{-3} \times 220} = 6.21 \text{ mm}$$

$$\approx 7 \text{ mm [Approx]}$$

$$\left[\because \text{Here, } K = 46.6 \times 10^{-3} \text{ kW/m/}^\circ\text{C for CI} \right]$$

$$(T_c - T_e) = 220^\circ\text{C for CI}$$

Q. Now, we should take the piston head with more thickness

As we compare both thicknesses, thickness of piston head based on the strength of piston material is higher than thickness based on heat dissipation.

$$\therefore \text{Thickness of piston head} = [t_1] = 16 \text{ mm.}$$

Step III: Thickness of rib $[t_2] = (0.3 \text{ to } 0.5) t_1 \rightarrow (\text{eq. 15.15})$
(Data Book)

$$\begin{aligned} &= 0.4 \times t_1 \\ &= 0.4 \times 16 \\ &= 6.4 \text{ mm} \\ &\approx 7 \text{ mm (Approx)} \end{aligned}$$

Step IV: Radial thickness of piston rings

$$\begin{aligned} [t_3] &= 0 \sqrt{\frac{3P_c}{\sigma_{bn}}} \rightarrow (\text{eq. 15.16}) (\text{Data Book}) \\ &= 100 \sqrt{\frac{3 \times 0.25}{90}} \\ &= 2.8 \text{ mm} \\ &\approx 3 \text{ mm (Approx)} \end{aligned}$$

Step V: Axial thickness of piston rings

$$\begin{aligned} [t_4] &= (0.7 - 1) t_3 \rightarrow (\text{eq. 15.17}) (\text{Data Book}) \\ &= 0.8 \times 3 \\ &= 2.4 \text{ mm} \\ &\approx 3 \text{ mm (Approx)} \end{aligned}$$

Step VI: Radial depth of ring-groove

$$\begin{aligned} b &= t_3 + 0.4 \text{ mm} \longrightarrow (\text{eq. 15.18}) (\text{Data Book}) \\ &= 3 + 0.4 \\ &= 3.4 \text{ mm} \\ &\approx 4 \text{ mm (Approx)} \end{aligned}$$

Step VII: Thickness of barrel nearest to piston head

$$\begin{aligned} t_5 &= 0.03D + b + 4.5 \text{ mm} \longrightarrow (\text{eq. 15.19}) (\text{Data Book}) \\ &= 0.03(100) + 4 + 4.5 \\ &= 11.5 \\ &\approx 12 \text{ mm (Approx)} \end{aligned}$$

Step VIII: Thickness of barrel at the open end of the piston

$$\begin{aligned} t_6 &= (0.25 \text{ to } 0.35) t_5 \longrightarrow (\text{eq. 15.20}) (\text{Data Book}) \\ &= 0.3 \times 12 = 3.45 \\ &\approx 4 \text{ mm (Approx)} \end{aligned}$$

Step IX:

Length of skirt $L_s = \frac{\pi H D P_m}{4 P_s} \longrightarrow (\text{eq. 15.21})$

$$\begin{aligned} &= \frac{\pi \times 0.1 \times 100 \times 5}{4 \times 0.5} = 78.53 \\ &\approx 79 \text{ mm} \end{aligned}$$

Step X:

Length of ring section $L_{r1} = i t_4 + (i-1) x_2 \longrightarrow (\text{eq. 15.22})$

$$\begin{aligned} &= 4(3) + (4-1)3 \\ &= 21 \text{ mm} \end{aligned}$$

$$\left[\begin{array}{l} \therefore i = 4 \\ x_2 \leq t_4 \\ x_2 = 3 \end{array} \right] \text{ from table (15.3)}$$

P XI :

Length of piston

$$L_p = L_s + L_n + L_t \longrightarrow \text{(eq. 15.23)}$$

$$= 79 + 21 + 16$$

$$= 116 \text{ mm}$$

$$\left[\text{where } L_t = x_1 = t_1 \right]$$

step XII :

Diameter of piston pin

$$d = \frac{F_g}{P_b \times l} \longrightarrow \text{(eq. 15.25)}$$

$$\Rightarrow F_g = \frac{\pi}{4} D^2 \cdot P_m$$

$$l = 1.5 d, \quad P_b = 25 \text{ N/mm}^2$$

$$= \frac{\pi}{4} (100)^2 \times 5$$

$d =$

$$= 39269.90 \text{ N}$$

$$d = \frac{F_g}{P_b \times l} = \frac{39269.90}{25 \times 1.5 d} = \sqrt{\frac{39269.90}{25 \times 1.5}}$$

$$= 32.36 \text{ mm}$$

step XIII :

Induced bending stress in piston pin

$$\sigma_b = \frac{32 M}{\pi d_i^3} < (\sigma_b) \longrightarrow \text{(eq. 15.26)}$$

$$M = \frac{F_g \cdot D}{8} = \frac{39269.90 \times 100}{8} = 490873.75$$

$$M = 490873.75$$

$$\sigma_b = \frac{32 M}{\pi d_i^3} = \frac{32 \times 490873.75}{\pi (32.36)^3} = 139.13 \text{ N/mm}^2$$

$$\text{Induced bending stress in piston pin } \sigma_b = 139.13 \text{ N/mm}^2 < (\sigma_b)$$

$\therefore$ our model is safe.

* Design a cast iron trunk type piston for a single acting engine developing 75 kW / cylinder when running at 600 rpm. The available data is maximum gas pressure 4.8 N/mm². Indicated Mean effective pressure 0.65 N/mm². Mechanical efficiency 95%. Radius of trunk 110 mm. Fuel consumption 0.3 kg / bph. calorific value of fuel is 44 × 10³ kJ/kg. Difference of temperature at the centre & edges of the piston head 200°C. Allowable stress for the material of the piston 33.5 MPa. Allowable stress for the material of piston rings & Gudgeon pin 80 MPa. Allowable bearing pressure on the piston barrel 0.4 N/mm². Allowable bearing pressure on the gudgeon pin 17 N/mm².

Solution:

Given data:

$$\text{Mass of fuel used } [m] = 0.3 \text{ kg / bph}$$

We need to convert into seconds

$$m = \frac{0.3}{3600} = 8.33 \times 10^{-5} \text{ kg / bph sec}$$

$$\text{speed of crank } [N] = 600 \text{ rpm}$$

$$\text{Maximum gas pressure } [P_m] = 4.8 \text{ N/mm}^2$$

$$\text{Effective pressure} = 0.65 \text{ N/mm}^2$$

$$\text{Radius of crank } [r] = 110 \text{ mm}$$

$$\text{calorific value of fuel } [C] = 44 \times 10^3 \text{ kJ/kg}$$

Step 1:

$$P_I = \frac{PLAN}{60 \times 10^6} \text{ kW} \longrightarrow (\text{eq. 15.14})$$

$$\text{stroke Length } [L] = 2 \times \text{radius}$$

$$= 2 \times 110 = 220 \text{ mm}$$

$$\Rightarrow 75 = \frac{0.65 \times 220 \times A \times 3600}{60 \times 10^6}$$

$$\Rightarrow 75 = \frac{0.65 \times 220 \times \frac{\pi}{4} D^2 \times 3600}{60 \times 10^6}$$

$$\therefore A = \frac{\pi}{4} D^2$$

$$D = \sqrt{\frac{75 \times 60 \times 10^6}{0.65 \times 220 \times 330 \times \frac{\pi}{4}}}$$

$$D = 365.45$$

$$D \approx 366 \text{ mm (Approx)}$$

step iii: Thickness of piston head based on the strength of piston material

$$t_1 = \sqrt{\frac{3 P_m D^2}{16 \sigma_{tp}}} \text{ mm} \longrightarrow \text{(eq. 15.13)}$$

$$= \sqrt{\frac{3 \times 4.8 \times (366)^2}{16 \times 35}} = 54.9 \text{ mm}$$

$$t_1 \approx 55 \text{ mm (Approx)}$$

step iv: Thickness of piston head based on heat dissipation

$$t_1 = \frac{1000 H}{12.56 K (T_c - T_e)} \text{ mm} \longrightarrow \text{(eq. 15.14)}$$

$$H = 0.05 m \times C_v \times P_B \text{ kW}$$

$$= 0.05 \times 8.34 \times 10^{-5} \times 44 \times 10^3 \times 71.25$$

$$= 13.05 \text{ kW}$$

$$t_1 = \frac{1000 H}{12.56 K (T_c - T_e)} \text{ mm}$$

$$= \frac{1000 \times 13.05}{12.56 \times 46.6 \times 10^{-3} \times 220}$$

$$= 101.34$$

$$t_1 \approx 102 \text{ mm (Approx)}$$

Thickness of piston ring based on heat dissipation greater than thickness of piston head based on strength of piston material.

$$\therefore \boxed{t_1 = 102 \text{ mm}}$$

step V:

Thickness of rib

$$t_2 = (0.3 - 0.5) t_1 \longrightarrow (\text{eq. 15.15})$$

$$t_2 = 0.4 \times 102$$

$$= 40.8 \text{ mm}$$

$$\boxed{t_2 \approx 41 \text{ mm}}$$

step VI:

Radial thickness of piston rings

$$t_3 = D \sqrt{\frac{3P_c}{\sigma_{br}}} \longrightarrow (\text{eq. 15.16})$$

$$= 366 \sqrt{\frac{3(0.025)}{90}}$$

$$\boxed{t_3 \approx 11 \text{ mm}}$$

step VII:

Axial thickness of piston rings

$$t_4 = (0.7 - 1) t_3 \longrightarrow (\text{eq. 15.17})$$

$$t_4 = 1 \times 11 = 11 \text{ mm}$$

$$\boxed{t_4 = 11 \text{ mm}}$$

iii: Thickness of barrel nearer to piston head

$$t_5 = 0.03D + b + 4.5 \text{ mm} \longrightarrow (\text{eq. 15.19})$$

$$b = t_3 + 0.4 \text{ mm} \longrightarrow (\text{eq. 15.18})$$

$$b = 11 + 0.4 \text{ mm}$$

$$= 11.4$$

$$b \approx 12 \text{ mm} \text{ Approx}$$

$$t_5 = 0.03(366) + 12 + 4.5$$

$$t_5 \approx 28 \text{ mm}$$

step ix: Thickness of barrel at the open end of the piston

$$t_6 = (0.25 \text{ to } 0.35) t_5 \longrightarrow (\text{eq. 15.20})$$

$$= 0.25 \times 28$$

$$t_6 = 7 \text{ mm}$$

step x: Length of skirt $L_s = \frac{\pi MDP_m}{4P_s} \longrightarrow (\text{eq. 15.21})$

$$L_s = \frac{\pi \times 0.03 \times 366 \times 4.8}{4 \times 0.28}$$

$$L_s = 275.95 \text{ mm}$$

step xi: Length of ring section $L_{r1} = it_4 + (i-1)x_2 \longrightarrow (\text{eq. 15.22})$

$$= 5(11) + (5-1)81 \quad x_2 \leq t_4$$

$$= 99 \text{ mm}$$

$$L_{r1} = 99 \text{ mm}$$

step xiii: Length of piston $L_p = L_s + L_n + L_t \longrightarrow$ (eq. 15.25)

$$= 215 + 102 + 99$$

$$= 416 \text{ mm}$$

Where $L_t = x_1 = t_1$

$$\boxed{L_p = 416 \text{ mm}}$$

step xiii: Diameter of piston pin $d = \frac{F_g}{P_b \times l} \longrightarrow$ (eq. 15.25)

$$\text{Where } F_g = \frac{\pi}{4} D^2 \cdot P_m$$

$$= 505002.22$$

$$\boxed{d = 140.7 \approx 141 \text{ mm}}$$

Induced bearing stress in piston pin

$$\sigma_b = \frac{32 M}{\pi d^3} < (\sigma_b) \longrightarrow \text{(eq. 15.26)}$$

$$\text{Where } M = \frac{F_g \cdot D}{8} = 23103851.57$$

$$\sigma_b = \frac{32 (23103851.57)}{\pi (141)^3}$$

$$\boxed{\sigma_b = 83.95 \text{ N/mm}^2} < (\sigma_b)$$

$\therefore$ our design is safe

Q. Design a connecting rod for an IC engine running at 1800 rpm & developing a maximum pressure of 3.15 N/mm^2 . The diameter of the piston is 100 mm. Mass of reciprocating parts for cylinder 2.25 kg. Length of connecting rod 380 mm, stroke of piston 190 mm. Take a factor of safety is 6 for design. Take length of diameter ratio for big end bearing as 1.3 & small end bearing as 2, & the corresponding bearing pressures as 10 N/mm^2 & 15 N/mm^2 , the density of material of the rod maybe taken as 8000 kg/m^3 & the allowable stress in the bolt as 60 N/mm^2 & in cap 80 N/mm^2 . The rod is to be I section for which you can choose your own proportions. Use Rankine formula for which the numerator constant maybe taken as 320 N/mm^2 & denominator constant $\frac{1}{7500} \text{ N/mm}^2$.

Solution:

We need to design a connecting rod

Given data:

Diameter of piston $[D] = 100 \text{ mm}$

Speed of IC engine $[n] = 1800 \text{ rpm}$

Weight of reciprocating parts $[W_r] = 2.25 \text{ kg}$

Factor of safety $[fs] = 6$

length of diameter of big end $\left[\frac{L_1}{d_1}\right] = 1.3$

Bearing pressure of big end $[P_{b1}] = 10 \text{ N/mm}^2$

length to diameter of small end $\left[\frac{L_2}{d_2}\right] = 2$

Bearing pressure of small end $[P_{b2}] = 15 \text{ N/mm}^2$

Maximum pressure = 3.15 N/mm^2

stroke Length $[L_s] = 190 \text{ mm}$

Allowable tensile stress of piston rod $[\sigma_t] = 60 \text{ N/mm}^2$

Allowable bending stress of cap of big end $[\sigma_{bc}] = 80 \text{ N/mm}^2$

Density of connecting rod material $[\rho] = 8000 \text{ kg/m}^3$

Allowable compressive stress $[\sigma_c] = 320 \text{ N/mm}^2$

Actual length of connecting rod $[L] = 380 \text{ mm}$

Step 1: Load due to steam or gas pressure

$$F_g = \frac{\pi}{4} D^2 p \longrightarrow (\text{eq. 16.1})$$

$$= \frac{\pi}{4} (100)^2 \times 3.15$$

$$F_g = 24740.04 \text{ N}$$

Step 2:

Design load

$$F_d = F_g \times f_g \longrightarrow (\text{eq. 16.8})$$

$$= 24740.04 \times 6$$

$$F_d = 148440.25 \text{ N}$$

Step 3:

Crippling load for connecting rod by using Rankin's formula

$$F_{cn} = \frac{\sigma_c \cdot A}{1 + a \left(\frac{L}{K} \right)^2} \geq F_d \longrightarrow (\text{eq. 16.4})$$

$$F_{cn} = F_d = 148440.25$$

$$\sigma_c = 320 \text{ N/mm}^2 \text{ (Given)}$$

$$A = 11 t^2 \text{ (from fig 16.3)}$$

$$a = \frac{1}{1500} \text{ (given)}$$

$$L = \lambda = 380 \text{ mm} \quad (\text{As both ends are hinged})$$

$$K = 1.78 t$$

$$\Rightarrow 148440.25 = \frac{320 \times 11 t^2}{1 + \frac{1}{7500} \left(\frac{380}{1.78 t} \right)^2}$$

$$\Rightarrow 148440.25 = \frac{3520 t^2}{1 + 1.3 \times 10^{-4} \left(\frac{4557.04}{t^2} \right)}$$

$$\Rightarrow 148440.25 = \frac{3520 t^2}{1 + \frac{5.94}{t^2}}$$

$$\Rightarrow 148440.25 = \frac{3520 t^2}{t^2 + 5.94}$$

$$(t^2 + 5.94) 148440.25 = 3520 t^2$$

$$3520 t^4 - 148440.25 t^2 = 881735.02$$

$$t^4 - 42.17 t^2 - 250.49 = 0$$

$$t = 6.89$$

$$\boxed{t = 7 \text{ mm}}$$

$$\text{Width} = 4t \quad (\text{from fig 16.3})$$

$$= 4(7) = 28 \text{ mm}$$

$$\text{Length} = 5t \quad (\text{from fig 16.3})$$

$$= 5(7)$$

$$= 35 \text{ mm}$$

Dimensions of big end & small end:

step 4: Diameter of piston pin = $d_1 = \frac{F_g}{P_{b1} \times d_1} \longrightarrow (\text{eq. 16.12})$

$$= \frac{24740.04}{10 \times 1.3 d_1}$$

$$d_1 = \sqrt{\frac{24740.04}{13}}$$

$$d_1 \approx 44 \text{ mm}$$

step 5:

Length of piston pin $l_1 = (1.8 \text{ to } 2) d_1 \rightarrow (\text{eq. 16.14})$

$$l_1 = 1.3 \times 44 = 56.6 \text{ mm}$$

$$l_1 = 57 \text{ mm}$$

step 6:

Diameter of crank pin $d_2 = \frac{F_g}{P_{b2} \times l_2} \rightarrow (\text{eq. 16.13})$

$$d_2 = \frac{24740.02}{15 \times 2 d_2}$$

$$d_2 = \sqrt{\frac{24740.02}{15 \times 2}}$$

$$d_2 = 29 \text{ mm}$$

step 7:

Length of crank pin $l_2 = (1 \text{ to } 1.25) d_2 \rightarrow (\text{eq. 16.15})$

$$l_2 = 2 d_2 = 2(29) = 58$$

$$l_2 = 58 \text{ mm}$$

step 8:

Inner diameter of small end $D_{is} = d_1 \rightarrow (\text{eq. 16.16})$

$$D_{is} = 44 \text{ mm}$$

step 9:

outer diameter of small end $D_{os} = d_1 + 2t_b + 2t_m \rightarrow (\text{eq. 16.17})$

$$= 44 + 2(4) + 2(10)$$

$$= 72 \text{ mm}$$

$$D_{os} = 72 \text{ mm}$$

step 10: Inner Diameter of big end $D_{ib} = d_2 \rightarrow (eq. 16.18)$

$$D_{ib} = d_2 = 29 \text{ mm}$$

step 11: outer diameter of big end $D_{ob} = d_2 + 2t_b + 2t_m \rightarrow (eq. 16.19)$

$$= 29 + 2(4) + 2(10)$$

$$D_{ob} = 57 \text{ mm}$$

step 12: Root diameter of bolt $d_b = 1.2 \times d_c \rightarrow (16.11)$

$$d_c = \left(\frac{2F_{im}}{\pi \sigma_t} \right)^{1/2} = \left(\frac{2 \times 187.5}{\pi \times 60} \right)^{1/2}$$

$$d_c = 2.9 \text{ mm}$$

$$d_c \approx 3 \text{ mm}$$

$$d_b = 1.2 \times 3$$

$$d_b = 3.6$$

$$d_b \approx 4 \text{ mm}$$

$$\text{Where } F_{im} = \frac{W_n}{g} \times \frac{\omega^2 r}{1000} \left[1 + \frac{r}{H/n} \right]$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1800}{60} = 188.4$$

$$r = \frac{L_s}{2} = \frac{190}{2} = 95 \text{ mm}$$

$$W_n = 2.25$$

$$F_{im} = \frac{2.25}{9.81} \times \frac{(188.4)^2 \times 95}{1000} \left[1 + \frac{1}{\frac{330}{95}} \right] = 187.15$$

step 13: Height of Big end $H_b = D_{ob} + 2d_b + 2t_m \rightarrow (eq. 16.20)$

$$= 57 + 2(4) + 2(10)$$

$$H_b = 85 \text{ mm}$$

step 14: Thickness of big end cap $t_c = \left(\frac{F_{im} \times d^1}{b \cdot \sigma_{bc}} \right)^{1/2} \rightarrow (eq. 16.21)$

$$\text{Where } d^1 = d_2 + 2t_b + d_b + 2t_m = 29 + 2(4) + 4 + 2(10) = 61 \text{ mm}$$

$$b = d_2 - 2t_b = 29 - 2(4) = 21 \text{ mm}$$

$$t_c = \left[\frac{787.56 \times 61}{50 \times 80} \right]^{1/2}$$

$$t_c = 12 \text{ mm}$$

step 15: Induced Maximum bending stress on the connecting rod

$$\sigma_b = \frac{M_{\max}}{Z} \longrightarrow \text{(eq. 16.19)}$$

$$\sigma_b = 28 \text{ N/mm}^2 \leq (\sigma_{bc})$$

$$M_{\max} = \frac{AP \omega^2 r J^2}{9\sqrt{3} \times 10^{12}} \text{ N-mm}$$

$$= \frac{11(71)^2 \times 8000 \times (188.4)^2 \times 95}{9\sqrt{3} \times 10^{12}} \left[\text{For "A" value refer table 16.3} \right]$$

$$= 134687.7 \text{ N-mm}$$

Section
Modulus

$$[Z] = 14t^3 \left[\text{from table 16.3 in page No. 16.9} \right]$$

$$= 14(71)^3$$

$$= 14(49)$$

$$Z = 4802$$

Determine the dimensions of dissection diesel engine to the particulars. Diameter of the piston is 110mm and stroke length is 140mm. Length of connecting rod is 420mm with speed 1500rpm. & rated rpm of engine is 1600. Mass of reciprocating parts of cylinder is 2.5kg. Maximum gas pressure is 4.5MPa and factor of safety is 7. Yield strength and compression strength is equal to 330MPa. Determine the whipping stresses.

Solution:

Given Data:

Piston Diameter $[D] = 110\text{mm}$

Stroke Length $[L] = 140\text{mm}$

Length of connecting rod $[l] = 420\text{mm}$

Speed $[N] = 1500\text{rpm}$

Rated speed of engine = 1600 rpm

Mass of reciprocating parts / cylinder $[m] = 2.5\text{kg}$

Maximum gas pressure = 4.5 MPa

Factor of safety $[fs] = 7$

Yield strength & compression strength = 330MPa

Step 1: Loss due to gas pressure

$$F_g = \frac{\pi}{4} D^2 p \longrightarrow \text{(eq. 10.1)}$$

$$= \frac{\pi}{4} (110)^2 \times 4.5$$

$$= 42764.93\text{N}$$

$$F_g = 42764.93\text{N}$$

step 2: Design Load

$$F_d = F_g \times f_s \longrightarrow (eq. 16.8)$$

$$= 42764.93 \times 7$$

$$= 299354.51$$

$$F_d = 299354.51$$

By using Rankine's formula we will find crippling load

step 3:

$$F_{cn} = \frac{\sigma_c \cdot A}{1 + a \left(\frac{L}{K} \right)^2} \geq F_d \longrightarrow (eq. 16.4)$$

$$\Rightarrow [F_{cn} = F_d]$$

$$299354.51 = \frac{330 \times 11t^2}{1 + \frac{1}{1500} \left[\frac{420}{1.78t} \right]^2} \quad \left["A = 11t^2" \text{ from fig 16.3} \right]$$

$$299354.51 = \frac{3630t^2}{1 + 1.33 \times 10^{-4} \left(\frac{176400}{3.1684t^2} \right)}$$

$$299354.51 = \frac{3630t^2}{1 + \frac{7.4047}{t^2}}$$

$$299354.51 (t^2 + 7.4047) = 3630t^4$$

$$3630t^4 - 299354.51t^2 = 2216630.34$$

$$\Rightarrow t^4 - 82.64t^2 - 610.64 = 0$$

$$t^2 = \frac{\pm 82.64 \pm \sqrt{(82.64)^2 - 4(-610.64)}}{2}$$

$$\Rightarrow t^2 = 89.29$$

$$\therefore t = 9.44$$

$$t \approx 10 \text{ mm}$$

width of connecting rod (w) = $4t$ $\rightarrow$ (from fig 16.3)

$$= 4t = 4(10)$$

$$= 40\text{mm}$$

Height of connecting rod (h) = $5t$ $\rightarrow$ (from fig 16.3)

$$= 5t = 5(10)$$

$$= 50\text{mm}$$

step 5: Diameter of piston pin $d_1 = \frac{F_g}{P_{b1} \times d_1}$ $\rightarrow$ (eq. 16.12)

$$d_1 = \frac{42764.93}{12.5 \times 1.5 d_1}$$

$$= 47.75$$

$$\boxed{d_1 \approx 48\text{mm}}$$

step 6: length of piston pin $l_1 = (1.5 \text{ to } 2) d_1$ $\rightarrow$ (eq. 16.14)

$$= 1.5 \times 48$$

$$\boxed{l_1 = 72\text{mm}}$$

step 7: Diameter of crank pin (d_2) = $\frac{F_g}{P_{b2} \times d_2}$ $\rightarrow$ (eq. 16.13)

$$= \frac{42764.93}{11 \times d_2}$$

$$= 62.35$$

$$\boxed{d_2 \approx 63\text{mm}}$$

Length of crank pin $l_2 = (1.0 \text{ to } 1.25) d_2$ $\rightarrow$ (eq. 16.15)

$$= 1 d_2 = 63$$

$$\boxed{l_2 = 63\text{mm}}$$

step 8: Innen diameter of small end $D_{is} = d_1 \longrightarrow$ (eq. 16.17)

$$D_{is} = 48 \text{ mm}$$

outer diameter of small end $D_{os} = d_1 + 2t_b + 2t_m \longrightarrow$ (eq. 16.18)

$$= 48 \text{ mm} + 2(4) + 2(10)$$

$$D_{os} = 76 \text{ mm}$$

$$\begin{cases} t_b = 2 \text{ to } 5 \text{ mm} \\ t_m = 5 \text{ to } 15 \text{ mm} \end{cases}$$

step 9: Innen diameter of Big end $D_{ib} = d_2 \longrightarrow$ (eq. 16.18)

$$D_{ib} = 63 \text{ mm}$$

outer diameter of Big end $D_{ob} = d_2 + 2t_b + 2t_m \longrightarrow$ (eq. 16.19)

$$= 63 + 2(4) + 2(10)$$

$$D_{ob} = 91 \text{ mm}$$

step 10: Root diameter of bolt $d_c = \left(\frac{2 F_{im}}{\pi \sigma_t} \right)^{1/2} \longrightarrow$ (eq. 16.11)

$$\Rightarrow F_{im} = \frac{W_n}{g} \times \frac{\omega^2 H}{1000} \left[1 + \frac{1}{(J/n)} \right] \longrightarrow$$

$$= \frac{2.5}{9.81} \times \frac{\left(\frac{2\pi \times 1500}{60} \right)^2 \times 70}{1000} \left[1 + \frac{1}{(420/70)} \right] = 511.82$$

$$\Rightarrow \sigma_t = \frac{\sigma_y}{f_s} = \frac{320}{7} = 47.14$$

$$\Rightarrow d_c = \left(\frac{2 F_{im}}{\pi \sigma_t} \right)^{1/2} = \left(\frac{2 (511.82)}{\pi (47.14)} \right)^{1/2} = 2.629$$

$$d_c \approx 3 \text{ mm}$$

$$d_b = 1.2 \times d_c \longrightarrow$$

$$= 1.2 \times 3$$

$$d_b \approx 4 \text{ mm}$$

Height of Big end

$$H_b = D_{ob} + 2d_b + 2t_m$$

$$= 91 + 2(4) + 2(10)$$

$$= 119 \text{ mm}$$

→ (eq. 16.20)

$$H_b = 119 \text{ mm}$$

∴ The values of d_b & t_m are taken from Data book pg. No. 16.6

Step 12:

Thickness of big-end cap

$$t_c = \left(\frac{F_{im} \times J'}{b \cdot \sigma_{bc}} \right)^{1/2} \rightarrow \text{(eq. 16.21)}$$

⇒ Distance between bolt centres of the big end

$$J' = d_2 + 2t_b + d_b + 2t_m$$

$$= 63 + 2(4) + 4 + 2(10)$$

$$\Rightarrow J' = 95 \text{ mm}$$

$$\Rightarrow b = d_2 - 2t_b = 63 - 2(4) = 63 - 8 = 55$$

$$\Rightarrow \sigma_{bc} = 100 \text{ N/mm}^2 \text{ (Assume)}$$

$$t_c = \left(\frac{F_{im} \times J'}{b \cdot \sigma_{bc}} \right)^{1/2} = \left(\frac{511.82 \times 95}{55 \times 100} \right)^{1/2} = 2.97$$

$$\therefore t_c \approx 3 \text{ mm}$$

Step 13:

Whipping stresses

$$\sigma_b = \frac{M_{max}}{Z} \rightarrow \text{(eq. 16.9)}$$

$$M_{max} = \frac{AP\omega^2 r J^2}{9\sqrt{3} \times 10^{12}} \text{ N-mm}$$

$$= \frac{10^2 (11) \times 7800 \times \left(\frac{2\pi \times 1500}{60} \right)^2 \times 70 \times 420}{9\sqrt{3} \times 10^{12}}$$

[Refer table 16.3]

$$= 399.27$$

$$Z = 14t^3 = 14 \times (10)^3 = 14000$$

$$\sigma_b = \frac{167695.14}{14000} = 11.978 \leq \sigma_{bc} \text{ i.e., } 100 \text{ N/mm}^2 \#$$

* Design a crank shaft which is overhung for the following
 Maximum load on the crank pin for maximum torque position is
 crank radius 200mm. Distance between crankpin centre & nearby bearing
 centre 300mm. Allowable stress in bending, shear & bearing are
 70MPa, 50MPa & 7MPa respectively.

Solution:

Given data

Force transmitted from connecting rod $[F] = 50 \text{ kN}$
 $= 50 \times 10^3 \text{ N}$

Distance b/w crankpin centre & nearby bearing centre $[x] = 300 \text{ mm}$

Radius of crank shaft $[r] = 200 \text{ mm}$

Allowable stresses:

Bending stress $(\sigma_b) = 70 \text{ MPa}$

Shear stress $(\tau) = 50 \text{ MPa}$

Bearing stress $(P_b) = 7 \text{ MPa}$

Step 1: Design of crank pin

$\Rightarrow$ From bearing stress

$$P_b = \frac{F}{1.2d} \leq (P_b) \longrightarrow (\text{eq. n.3})$$

$$\Rightarrow 7 = \frac{50 \times 10^3}{1.2d \times d}$$

$$\Rightarrow d = 77.15 \approx 78 \text{ mm}$$

$$d = 78 \text{ mm}$$

$$l = 1.2 \times d = 1.2 (78) = 93.6$$

$$l = 94 \text{ mm}$$

$$\tau = \frac{4F}{\pi d^2} \leq (\tau) \longrightarrow (\text{eq. 17.1})$$

$$= \frac{4 \times 50 \times 10^3}{\pi (78)^2}$$

$$\tau = 10.46$$

$\tau = 10.46$ which is less than allowable shear stress i.e., 50 MPa

$\Rightarrow$ From bending stress

$$\sigma_b = \frac{16FJ}{\pi d^3} \leq (\sigma_b) \longrightarrow (\text{eq. 17.2})$$

$$= \frac{16 \times 50 \times 10^3 \times 94}{\pi \times 78^3} = 50.44$$

$$\sigma_b = 50.44$$

Step 2: Design of crank shaft journal

$$D = (1.25 \text{ to } 1.5) d = 1.25 \times 78 = 97.5$$

$$D \approx 98$$

$$L = 1.2D = 1.25 \times 98 = 122.5$$

$$L \approx 123$$

As we assume $(\theta + \phi) = 90^\circ$

Radial component of force $F_H = F \cos(\theta + \phi) \longrightarrow (\text{eq. 17.14})$

$$F_H = F \cos 90^\circ$$

$$F_H = 0$$

Tangential component of force $F_T = F \sin(\theta + \phi) \longrightarrow (\text{eq. 17.15})$

$$F_T = F \sin 90^\circ = F$$

$$F_T = 50 \times 10^3 \text{ N}$$

Lct us assume $D = 140 \text{ mm}$

⇒ Bending stress by tangential component

$$\begin{aligned}\sigma_{bt} &= \frac{32 F_t \cdot x}{\pi D^3} \leq (\sigma_b) \longrightarrow (\text{eq. 17.9}) \\ &= \frac{32 \times 50 \times 10^3 \times 300}{\pi \times 140^3} \\ &= 55.68 < (\sigma_b)\end{aligned}$$

$\sigma_{bt} = 55.68$ which is less than Allowable bending stress i.e., 70 MPa

⇒ Resultant bending stress

$$\begin{aligned}\sigma_b &= \frac{32 F \cdot x}{\pi D^3} \leq (\sigma_b) \longrightarrow (\text{eq. 17.10}) \\ &= \frac{32 \times 50 \times 10^3 \times 300}{\pi \times 140^3} \\ &= 55.68 < (\sigma_b) = 70 \text{ MPa}\end{aligned}$$

⇒ Torsional shear stress due to tangential force

$$\begin{aligned}\tau &= \frac{16 F_t \cdot x}{\pi D^3} \leq (\tau) \longrightarrow (\text{eq. 17.11}) \\ &= \frac{16 \times 50 \times 10^3 \times 300}{\pi (140)^3} \\ &= 18.56 < (\tau = 50 \text{ MPa})\end{aligned}$$

⇒ Equivalent bending stress

$$\begin{aligned}\sigma_{be} &= \frac{1}{2} \left[\sigma_b + \sqrt{\sigma_b^2 + 4\tau^2} \right] \leq (\sigma_b) \longrightarrow (\text{eq. 17.12}) \\ &= \frac{1}{2} \left[55.68 + \sqrt{(55.68)^2 + 4(18.56)^2} \right] \\ &= 61.29 < (\sigma_b) = 70 \text{ MPa}\end{aligned}$$

$$\boxed{\sigma_{be} = 61.29 \text{ MPa}}$$

A crank shaft of single throw centre crank type is to be designed for a diesel engine. The crank shaft is carrying one fly wheel at one end of each journal. The other specifications are diameter of piston 200mm, stroke of piston 300mm, length of connecting rod 600mm, weight of each fly wheel 2kN, distance between flywheel & its nearest journal 150mm, speed of the engine 1000 rpm. Maximum gas pressure is 1 N/mm^2 . Maximum torque is experienced in the crankshaft in the crank angle is 30° from inner dead centre position. Allowable stresses for the crank shaft material are in bending 70 N/mm^2 , in shear 40 N/mm^2 , in bearing 8 N/mm^2 . Design the crank shaft & give a neat sketch.

Solution:

Given data:

Diameter of piston = 200mm

Length of stroke (L_s) = 300mm

Length of connecting rod (L_c) = 600mm

Weight of each fly wheel (W) = 2kN

Distance b/w flywheel & its nearest journal (r_f) = 150mm

speed of engine (N) = 1000 rpm

Maximum gas pressure (P_{max}) = 1 N/mm^2

crank angle (θ) = 30°

Allowable stresses:

Bending stress (σ_b) = 70 N/mm^2

Shear stress (τ) = 40 N/mm^2

Bearing stress (p_b) = 8 N/mm^2

⇒ Equivalent shear stress

$$\begin{aligned}\tau_e &= \frac{1}{2} \sqrt{\sigma_b^2 + 4\tau^2} < (\tau) \longrightarrow (\text{eq. n. 13}) \\ &= \frac{1}{2} \sqrt{(55.68)^2 + 4(18.56)^2} \\ &= 33.45 < (\tau = 50 \text{ Mpa})\end{aligned}$$

$$\boxed{\tau_e = 33.45}$$

Step 3: Design of crank web

$$t = (0.7 \text{ to } 1.0) d \longrightarrow [\text{from data book pg. 11.5}]$$

$$\text{Width of crank web } w = \frac{a+b}{2} \longrightarrow (\text{eq. n. 17})$$

$$a = 1.5 d = 1.5 \times 78 = 117 \text{ mm}$$

$$b = 1.5 D = 1.5 \times 140 = 210 \text{ mm}$$

$$w = \frac{a+b}{2} = \frac{117 + 210}{2} = \frac{327}{2} = 163.5$$

$$\boxed{w \approx 164 \text{ mm}}$$

⇒ Bending stress By tangential force

$$\sigma_{bt} = \frac{6 F_t \cdot r_1}{t w^2} \leq (\sigma_b) \longrightarrow (\text{eq. n. 6})$$

$$= \frac{6 \times 50 \times 10^3 \times 200}{71 \times (164)^2}$$

$$\sigma_{bt} = 31.41 < (\sigma_b = 70 \text{ Mpa})$$

⇒ overall stress acting on the web

$$\sigma = \sigma_a + \sigma_{bt} + \sigma_{te} \longrightarrow (\text{eq. 17.11})$$

$$= 0 + 0 + 31.41$$

$$\boxed{\sigma = 31.41} \leq (\sigma_b) 70 \text{ Mpa}$$

$$\sigma_b = \frac{8F_x}{\pi d^3}$$

$$= \frac{8 \times 31415.91 \times 250}{\pi \times 58^3}$$

$$\sigma_b = 102.50 > (\sigma_b = 70)$$

take $d = 70$

[change "d" value to get less value of σ_b]

$$L = 1.25 \times 70$$

$$= 87.5$$

$$L \approx 88 \text{ mm}$$

$$t = 0.9 \times 70$$

$$t = 63 \text{ mm}$$

$$d = 1.2 \times 70$$

$$d = 84 \text{ mm}$$

$$X = d + 2t + L = 88 + 2(63) + 84 = 298$$

$$\sigma_b = \frac{8 \times 31415.91 \times 298}{\pi \times (70)^3} = 69.50$$

$$\sigma_b = 69.50 < (\sigma_b = 70)$$

step 2:

Design of crank shaft journal

$$D = d = 70 \text{ mm}$$

$$L = 1.25 \times D = 1.25 \times d = 1.25 \times 70$$

$$L = 88 \text{ mm}$$

⇒ Resultant bending stress

$$\sigma_b = \frac{32 W.Y}{\pi D^3} \rightarrow \text{eq. 17.10}$$

$$= \frac{32 \times 2 \times 10^3 \times 50}{\pi \times 70^3}$$

$$= 8.9 < (\sigma_b = 70)$$

Step 1: Design of crank pin

⇒ From bearing stress $P_b = \frac{F}{ld} \leq (P_b) \longrightarrow \text{(eq. 17.1)}$

$$\text{Force } F = \frac{\pi}{4} D^2 p$$

$$= \frac{\pi}{4} (200)^2 \times 1$$

$$= 31415.91$$

$$P_b = \frac{F}{ld}$$

$$\Rightarrow 8 = \frac{31415.91}{1.2d \times d}$$

$$\Rightarrow d = 57.80$$

$$\boxed{d = 58 \text{ mm}}$$

$$l = 1.2 \times d = 1.2 \times 58 = 69.6$$

$$\boxed{l \approx 70 \text{ mm}}$$

⇒ From plain on shear stress $\tau = \frac{2F}{\pi d^2} \leq (\tau) \longrightarrow \text{(eq. 17.1)}$

$$= \frac{2 \times 31415.91}{\pi \times 58^2}$$

$$\tau = 5.94 < (\tau = 40 \text{ N/mm}^2)$$

⇒ Bending stress

$$\sigma_b = \frac{8Fr}{\pi d^3} \leq (\sigma_b) \longrightarrow \text{(eq. 17.2)}$$

Distance b/w centres of

crank
shaft

$$x = L + 2t + d$$

Not given in the
Data Book

$$L = 1.25 D = 1.2 \times 58 = 72.5 \approx 73$$

$$D = d = 58$$

$$t = (0.7 \text{ to } 1.0) d = 0.9 \times 58 = 52.2 \approx 53$$

$$x = 73 + 2(53) + 70 = 249 \approx 250$$

Torsional shear stress due to tangential force

$$\tau = \frac{8 F_n \cdot H}{\pi D^3} \leq (\tau) \rightarrow (eq. 17.11)$$

$$F_n = F \cos(\theta + \phi) \rightarrow (eq. 17.14)$$

$$\frac{L_c}{\sin \theta} = \frac{H}{\sin \phi}$$

$$\frac{600}{\sin 30} = \frac{150}{\sin \phi}$$

$$\sin \phi = \frac{150 \times \sin 30}{600}$$

$$\phi = \sin^{-1} \left[\frac{150 \times \sin 30}{600} \right] = 37.180$$

$$F_n = F \cos(37.180)$$

$$= 31415.9 \times \cos(37.180)$$

$$F_n = 25030.33$$

$$F_t = F \sin(\theta + \phi) \rightarrow (eq. 17.15)$$

$$= 31415.91 \times \sin(37.180)$$

$$F_t = 18985.28$$

$$\tau = \frac{8 F_n \cdot H}{\pi D^3} = \frac{8 \times 25030.33 \times 150}{\pi \times (70)^3}$$

$$\tau = 27.87 < (\tau = 40)$$

$\Rightarrow$ Equivalent bending stress $\sigma_{bc} = \frac{1}{2} [\sigma_b + \sqrt{\sigma_b^2 + 4\tau^2}] \rightarrow (eq. 17.12)$

$$= \frac{1}{2} [69.50 + \sqrt{69.50^2 + 4(27.87)^2}]$$

$$\sigma_{bc} = 44.54$$

Design of crank shaft journal

$$D = 65 \text{ mm}$$

$$\sigma_b = \frac{32 W Y}{\pi D^3} \longrightarrow (eq. 17.10)$$

$$= \frac{32 \times 2 \times 10^3 \times 150}{\pi \times (65)^3}$$

$$\boxed{\sigma_b = 11.12}$$

⇒ Equivalent shear stress

$$\tau_c = \frac{1}{2} \sqrt{\sigma_b^2 + 4\tau^2} \longrightarrow (eq. 17.13)$$

$$= \frac{1}{2} \sqrt{(11.12)^2 + 4(34.81)^2}$$

$$= 35.25 < (\tau = 40)$$

step 3: Design of crank web

$$W = 1.5 \times d$$

$$= 1.5 \times 70$$

$$= 105$$

$$t = (0.7 \text{ to } 1.0) d$$

$$= 0.9 \times 70$$

$$= 63 \text{ mm}$$

⇒ Axial stress by radial force

$$\sigma_a = \frac{F_r}{2W \cdot t} \leq (\sigma_b) \longrightarrow (eq. 17.14)$$

$$= \frac{25030.33}{2 \times 105 \times 63}$$

$$\boxed{\sigma_a = 1.59}$$

⇒ Bending stress by radial force $\sigma_{br} = \frac{3F_r [x - (u+t)]}{2wt^2} \leq (\sigma_b) \rightarrow (eq. 17.5)$

$$= \frac{3 \times 25030.33 [298 - (84+63)]}{2 \times 105 \times 63^2}$$

$$\boxed{\sigma_{br} = 13.60}$$

⇒ Bending stress by tangential force $\sigma_{bt} = \frac{6F_t \cdot r}{2wt^2} \leq (\sigma_b) \rightarrow (eq. 17.6)$

$$= \frac{6 \times 18985.28 \times 150}{2 \times 63 \times 105^2}$$

$$\boxed{\sigma_{bt} = 12.30}$$

⇒ overall stress acting on web

$$\sigma = \sigma_a + \sigma_{br} + \sigma_{bt} \leq (\sigma_b) \rightarrow (eq. 17.7)$$

$$= 1.89 + 13.60 + 12.30$$

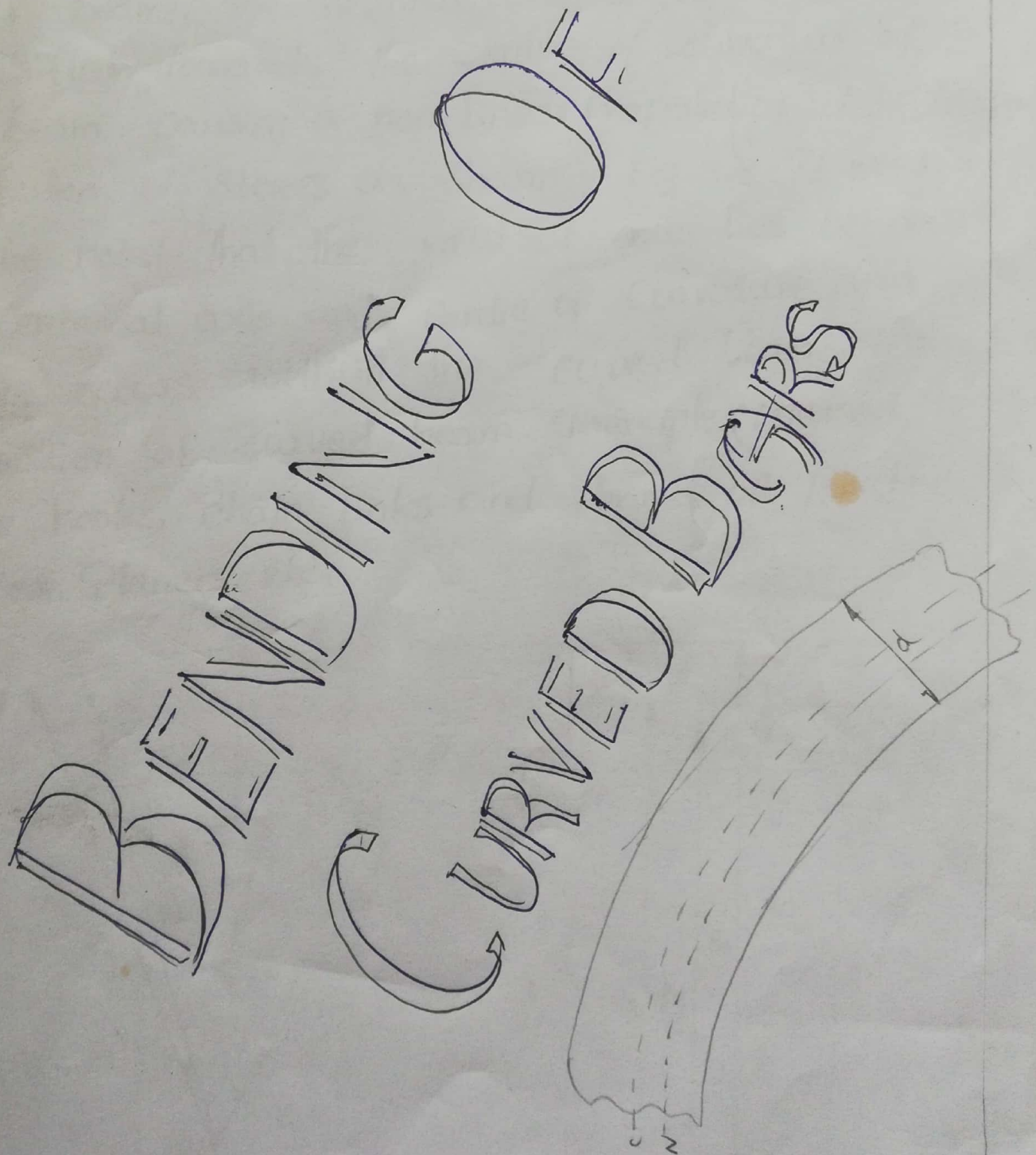
$$\boxed{\sigma = 27.791} < (\sigma_b = 90)$$

Hence all the induced stresses are less than all allowable stresses.

∴ our design is safe.

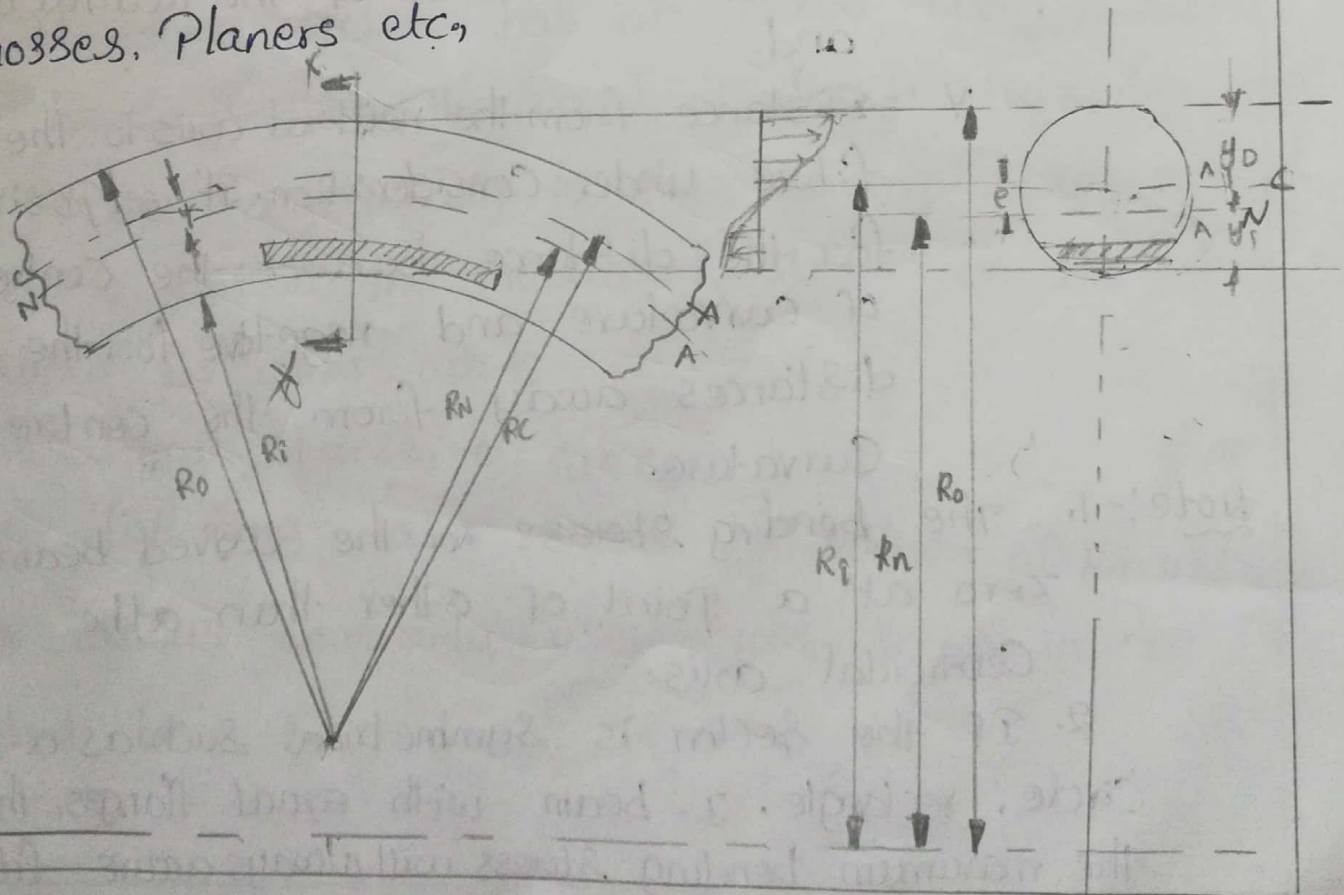
UNIT-III

CURVED BEAMS



Bending Stress in Curved Beams:

We have seen in the Previous article that for the straight beams, the neutral axis of the section coincides with its centroidal axis. The stress distribution in the beam is linear. But in case of curved beams, the neutral axis of the cross-section is shifted towards the centre of curvature of the beam causing a non-linear (hyperbolic) distribution of stress, as shown in Fig. 5.8. It may be noted that the neutral axis lies between the centroidal axis and centre of curvature and always occurs within the curved beams. The application of curved beam principle is used to crane hooks, chain links and frames of punches, presses, planers etc.



Consider a curved beam subjected to a bending moment M , as shown in Fig. 5.8. In finding the bending stress in curved beams the same assumptions are used as for straight beams. The general expression for the bending stress (σ_b) in a curved beam at any fibre at the distance y from the neutral axis is given by

$$\sigma_b = \frac{M}{A \cdot e} \left(\frac{y}{R_n - y} \right)$$

Where M = Bending moment acting the given section about the centroidal axis.

A = Area of Crossing Section

e = Distance from the centroidal axis to the ^{neutral} ~~centroidal~~ axis = $R - R_n$

R = Radius of Curvature of the Centroidal axis.

R_n = Radius of curvature of the neutral axis and

y = Distance from the neutral axis to the fibre under consideration. It is positive for the distance towards the centre of curvature and negative for the distances away from the centre of curvature.

Note:- 1. The bending stress in the curved beam is zero at a point other than at the Centroidal axis

2. If the section is symmetrical such as a circle, rectangle, I-beam with equal flanges, then the maximum bending stress will always occur fibre.

3. If the section is symmetrical, then the maximum bending stress may occur at the either the inside fibre or the outside fibre. The maximum bending stress at the inside fibre is given by

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i}$$

Where

y_i : Distance from the neutral axis to the inside fibre = $R_o - R_i$ and

R_o : Radius of curvature of the outside fibre

It may be noted that the bending stress at the inside fibre is tensile while the bending stress at the outside fibre is compressive.

4. If the section has an axial load in addition to bending, then the axial or direct stress (σ_d) must be added algebraically to the bending stress, in order to obtain the resultant stress on the section. In other words

Resultant stress, $\sigma = \sigma_d \pm \sigma_b$

The following table shows the values of R_o and R_i for various commonly used cross-section in the curved beam.

Introduction:-

In chapter 5 being education $\left(\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}\right)$ was derived assuming the beam to be initially straight (besides other fundamental assumption) before the application of bending movement. However, machine members and structures subjected to bending are not always straight, as in the case of car hooks, chain links etc., before a bending moment is applied to them. The simple flexure formula may be used for curved beams of which radius of curvature is more than five times the depth of beam. The simple bending formula, however is not applicable for deeply curved beams where the neutral and centroidal axes do not coincide. To deal with such cases Winkler-Bach Theory is used.

STRESSES IN CURVED BARS (Winkler-Bach Theory)

Fig 20.1 shows a bar ABCD initially in its unstrained state. Let A'B'CD be the strained position of the bar.

Let, R = Radius of Curvature of the centroidal axis H G.

y : Distance of fibre EF from the centroidal layer H G.

R_1 : Radius of curvature of HG_1'

y_1 : Distance between the EF' and HG_1' after straining

M : Uniform bending moment applied to the beam (assumed positive when bending to increase curvature)

θ : Original angle subtended by centroidal axis HG at its centre of curvature O , and

θ' : Angle subtended by HG_1' at its centre of curvature

The following assumptions are made in this analysis

1. Plan section (transverse) remains plan during bending
2. The material obeys Hooke's law (limit of proportionality is not exceeded)
3. Radial strain is negligible
4. The fibres are free to expand or contract without any constraining effect from the adjacent fibres.

For finding the strain and stress normal to the section

Consider the fibre EF at a distance y from the Centroidal axis.

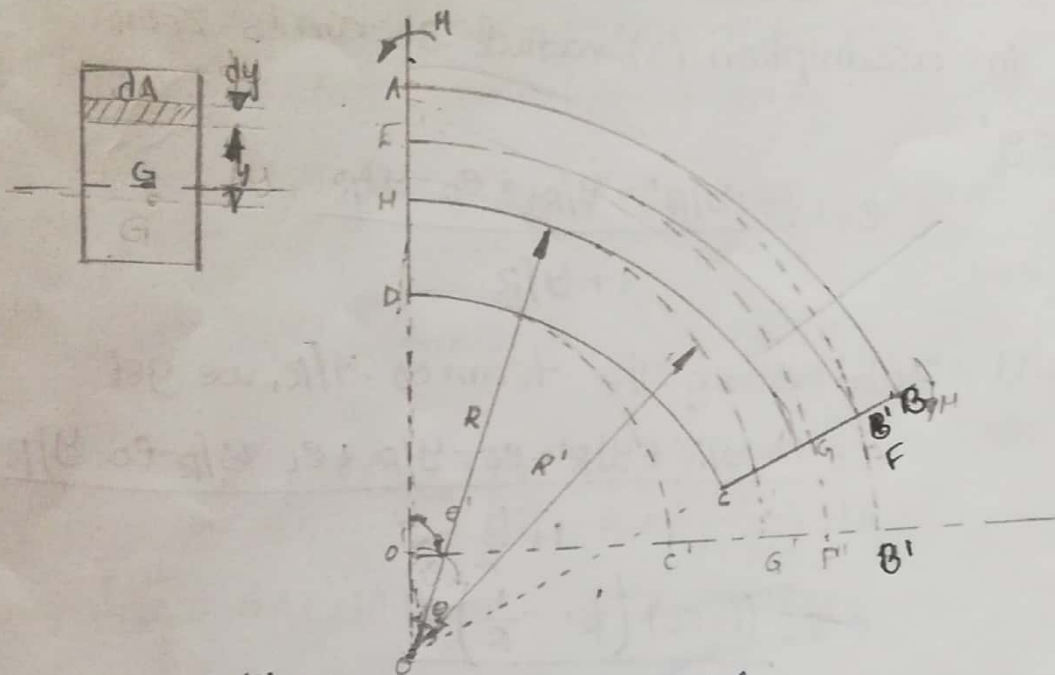


Fig 20.1 Bending Curved bar

Let be the stress in the strained layer EF' under the bending moment M and e is strain in the same layer

$$\text{Strain } e = \frac{EF' - EF}{EF} = \frac{(R' + y')\theta' - (R + y)\theta}{(R + y)\theta} \quad \text{or, } e = \frac{R' + y'}{R + y} \frac{\theta'}{\theta} - 1$$

Also e_0 = strain in the Centroidal layer i.e., when $y = 0$

$$= \frac{R'\theta'}{R\theta} - 1 \quad \text{or} \quad 1 + e = \frac{R' + y'}{R + y} \frac{\theta'}{\theta}$$

and $1 + e_0 = \frac{R'}{R} \cdot \frac{\theta'}{\theta}$

Dividing equation (20.1) and (20.2) we get

$$\frac{1 + e}{1 + e_0} = \frac{R' + y'}{R + y} \cdot \frac{R}{R'} = \frac{1 + y'/R}{1 + y/R}$$

or $e = (1 + e_0) \left[\frac{1 + y'/R}{1 + y/R} \right] - 1$

or $e = \frac{e_0 - y/R + y/R' + e_0 - y/R}{1 + y/R}$

According to assumption (3), radius strain is zero

$$\therefore y = y'$$

Strain,
$$e = \frac{e_0 \cdot y/R' + y/R' + e_0 - y/R}{1 + y/R}$$

Adding and subtracting the term $e_0 \cdot y/R$, we get

$$e = \frac{e_0 \cdot y/R' + y/R' + e_0 - y/R + e_0 \cdot y/R - e_0 \cdot y/R}{1 + \frac{y}{R}}$$

or
$$e = \frac{(1+e_0) \left(\frac{1}{R'} - \frac{1}{R} \right) y}{1 + \frac{y}{R}}$$

From the Fig. 20.1 it's obvious that for the given bending moment the layers above the Centroidal layer are in tension and layer below the Centroidal layer in Compression

Stress,
$$\sigma = Ee = E \left[e_0 + \frac{(1+e_0) \left(\frac{1}{R'} - \frac{1}{R} \right) y}{\left(1 + \frac{y}{R} \right)} \right]$$

(where, E = Young's modulus of the material)

Total force on the section $F = \int \sigma \cdot dA$

Considering a small strip of elementary area dA , at a distance of y from the centroidal layer

HG, we have

$$\begin{aligned} F &= E \int e_0 \cdot dA + E \int \frac{(1+e_0) \left(\frac{1}{R'} - \frac{1}{R} \right) y \cdot dA}{1 + y/R} \\ &= E \int e_0 \cdot dA + E (1+e_0) \left(\frac{1}{R'} - \frac{1}{R} \right) \int \frac{y}{1 + y/R} dA \end{aligned}$$

or
$$F = E e_0 \cdot A + E (1+e_0) \left(\frac{1}{R'} - \frac{1}{R} \right) \int \frac{y}{1 + y/R} dA.$$

where A = area of cross-section of the beam

The total resisting moment is given by

$$M = \int \sigma \cdot y \cdot dA = E \int e_0 \cdot y \cdot dA + E \int \frac{(1+e_0) \left(\frac{1}{R_1} - \frac{1}{R} \right)}{1+y/R} \cdot y^2$$

$$: E \cdot e_0 \times 0 = E (1+e_0) \left(\frac{1}{R_1} - \frac{1}{R} \right) \int \frac{y^2}{1+y/R} dA \quad [\because \int y dA = 0]$$

$$\therefore M = E (1+e_0) \left(\frac{1}{R_1} - \frac{1}{R} \right) \int \frac{y^2}{1+y/R} dA$$

$$\text{Let, } \int \frac{y^2}{1+y/R} dA = Ah^2$$

Where, h^2 = a constant for the cross-section of the beam

$$\therefore M = E (1+e_0) \left(\frac{1}{R_1} - \frac{1}{R} \right) Ah^2$$

$$\begin{aligned} \text{Now, } \int \frac{y}{1+y/R} dA &= \int \left(y - \frac{y^2}{R+y} \right) dA = \int y dA - \int \frac{y^2}{R+y} dA \\ &= 0 - \frac{1}{R} \int \frac{y^2}{1+y/R} dA \end{aligned}$$

$$\therefore \int \frac{y}{1+y/R} dA = -\frac{1}{R} \int \frac{y^2}{1+y/R} dA = -\frac{1}{R} Ah^2$$

Hence eqn (20.5) becomes

$$F = E e_0 \cdot A - E (1+e_0) \left(\frac{1}{R_1} - \frac{1}{R} \right) \frac{Ah^2}{R}$$

Since transversal plane section remains plane during bending

$$F = 0$$

$$\text{or, } 0 = E e_0 \cdot A - E (1+e_0) \left(\frac{1}{R_1} - \frac{1}{R} \right) \frac{Ah^2}{R}$$

$$\text{or } E e_0 \cdot A = E (1+e_0) \left(\frac{1}{R_1} - \frac{1}{R} \right) \frac{Ah^2}{R}$$

or.

$$e_0 = E (1 + e_0) \left(\frac{1}{R'} - \frac{1}{R} \right) \frac{Ah^2}{R}$$

or

$$\frac{e_0 R}{h^2} = (1 + e_0) \left(\frac{1}{R'} - \frac{1}{R} \right)$$

Substituting the value of $(1 + e_0) \left(\frac{1}{R'} - \frac{1}{R} \right)$ in eqn (20.7), we get

$$M = E \cdot \frac{e_0 R}{h^2} \cdot Ah^2 = e_0 EAR$$

or

$$e_0 = \frac{M}{EAR}$$

Substituting the value of e_0 in eqn (20.4) we get

$$\sigma = \frac{M}{AR} + E \times \left(\frac{y}{1 + y/R} \right) \times \frac{e_0 R}{h^2}$$

$$= \frac{M}{AR} + E \times \frac{y}{1 + y/R} \times \frac{R}{h^2} \times \frac{M}{EAR} \quad \text{Substing } e_0 = \frac{M}{EAR} \text{ again}$$

$$= \frac{M}{AR} + \frac{MR}{AR} \times \frac{Ry}{1 + y/R} \times \frac{1}{h^2}$$

$$\sigma = \frac{M}{AR} \left[1 + \frac{R^2}{h^2} \times \left(\frac{y}{R + y} \right) \right]$$

On the other side of HG, y will be negative and stress will be compressive

$$\therefore \sigma = \frac{M}{AR} \left[1 - \frac{R^2}{h^2} \left(\frac{y}{R - y} \right) \right]$$

When the bending moment applied in such a manner that it tends to decrease the curvature then eqn. (20.12) will give compressive stress and eqn. (20.13) tensile stress

Position of neutral axis:-

At the neutral axis, $\sigma = 0$

$$\therefore \frac{M}{AR} \left[1 + \frac{R^2}{h^2} \left(\frac{y}{R + y} \right) \right] = 0$$

or.

$$R^2/h^2 (y/R + y) = -1$$

$$R^2 y = -h^2 (R + y) = -Rh^2 - h^2 y$$

$$\text{or, } y(R^2 + h^2) = -Rh^2$$

$$y = \frac{-Rh^2}{R^2 + h^2}$$

Since the neutral axis is located below the centroidal axis.

VALUES OF h^2 FOR VARIOUS SECTIONS

We know

$$h^2 = \frac{1}{A} \int \frac{y^2}{1+y/R} \cdot dA$$

$$= \frac{R}{A} \int \frac{y^2}{R+y} \cdot dA$$

$$= \frac{R}{A} \left[\int y dA - \int R dA + \int \frac{R^2 dA}{R+y} \right]$$

$$= \frac{R}{A} \left[0 - RA + \int \frac{R^2 dA}{R+y} \right]$$

$$h^2 = \frac{R^3}{A} \int \frac{dA}{R+y} - R^2$$

Rectangular Section:-

Fig 20.2 shows the rectangular section with Centre of Curvature O laying on YY-axis. xx-axis is the centroidal bending axis. Consider an elementary strip of width B and depth dy at a distance y from the centroidal layer.

Area of Strip, $dA = B \cdot dy$

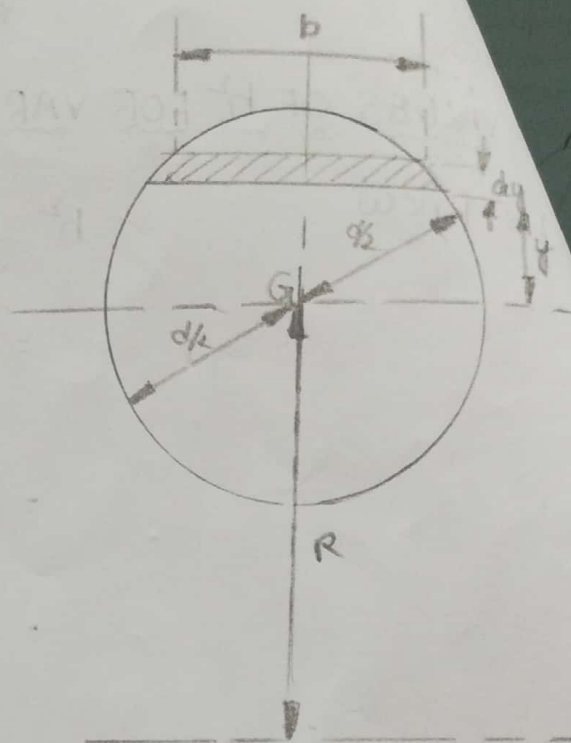
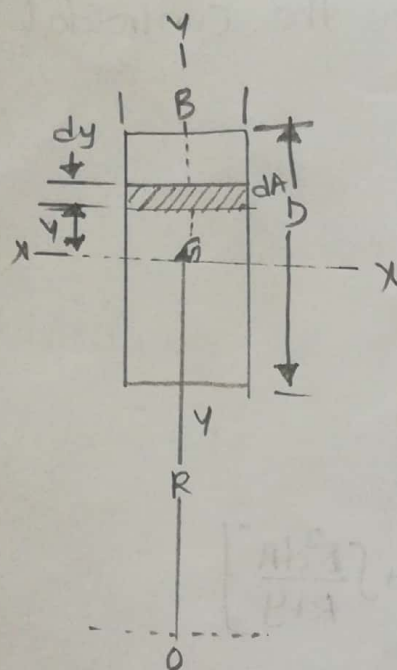
$\therefore$

$$A = B \times D$$

$$h^2 = \frac{R^3}{B \times D} \int_{-D/2}^{+D/2} \frac{B \cdot dy}{R+y} - R^2$$

$$= \frac{R^3}{D} \left[\log_e (R+y) \right]_{-D/2}^{+D/2} - R^2$$

or,
$$h^2 = \frac{R^3}{D} \log_e \left(\frac{2R+D}{2R-D} \right) - R^2$$



Circular Section:-

Fig 20.3 Shows the circular section of ^{distance of} diameter d of a curved bar of radius of curvature R , from the centre of curvature O up to the centroid G of the section.

Area of Cross Section, $A = \frac{\pi}{4} d^2$

Consider a strip of width b and a depth dy at a distance y from the centroidal layer as shown

$$b = 2\sqrt{\left(\frac{d}{2}\right)^2 - y^2} \cdot dy$$

Area of Strip $dA = b \cdot dy$

$$= 2\sqrt{\left(\frac{d}{2}\right)^2 - y^2} \cdot dy$$

$$h^2 = \frac{R^3}{A} \int_{-d/2}^{+d/2} \frac{2\sqrt{\left(\frac{d}{2}\right)^2 - y^2}}{R+y} \cdot dy - R^2$$

$$= \frac{8R^3}{\pi d^2} \int_{-d/2}^{+d/2} \frac{\frac{d}{4} - y^2}{R+y} \cdot dy - R^2$$

Expanding the integral by binomial expression and then integrating $h^2 = \frac{d^2}{16} + \frac{1}{128} \frac{d^4}{R^2} + \dots$

Triangular Section:-

Refer to Fig. 20.4

$$\text{Let } R+y=a$$

$$dy = da$$

Width of elementary strip,

$$b' = \left(\frac{R_2 - a}{d} \right) b$$

Area of the elementary

Strip

$$dA = b' \cdot dy = b' \cdot da$$

$$\text{Now, } h^2 = \frac{R^3}{A} \int_{R_1}^{R_2} \frac{dA}{R+y} - R^2$$

$$= \frac{R^3}{A} \int_{R_1}^{R_2} \left(\frac{R_2 - a}{d} \right) b \cdot \frac{da}{a} - R^2$$

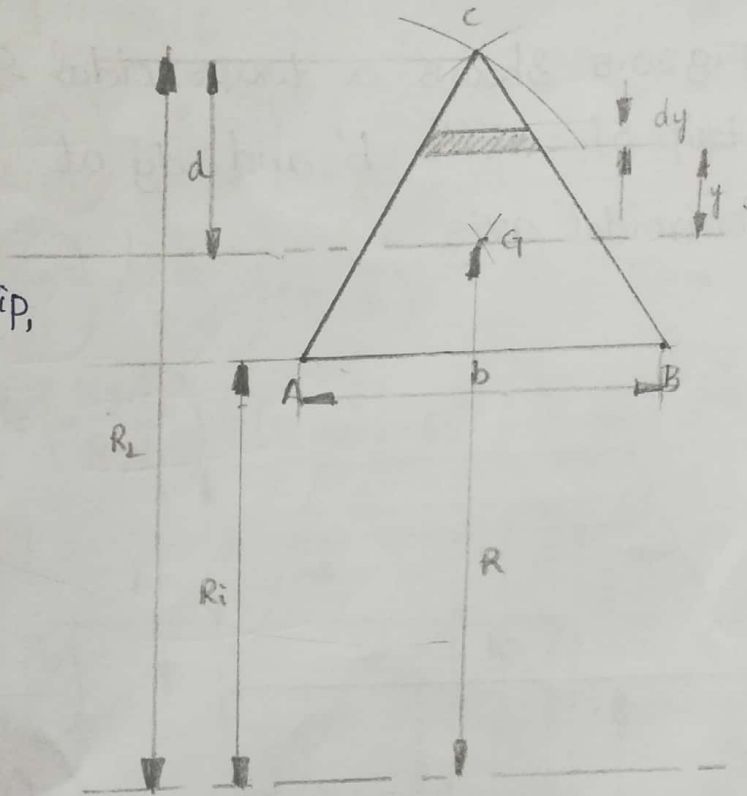
$$= \frac{R^3}{A} \left[\frac{R_2 b}{d} \log_e \frac{R_2}{R_1} - \frac{b}{d} (R_2 - R_1) \right] - R^2$$

$$= \frac{R^3}{A} \left[\left(d + \frac{2h}{3} \right) \frac{b}{d} \cdot \log_e \left(\frac{3R+2d}{3R-d} \right) - b \right] - R^2$$

$$\left[\begin{array}{l} \therefore R_2 = R + \frac{d}{3}, \text{ and} \\ R_1 = R - \frac{d}{3}, \text{ and} \\ R_2 - R_1 = d \end{array} \right]$$

$$= \frac{2R^3}{bd} \left[(3R+2d) \frac{b}{3d} \cdot \log_e \left(\frac{3R+2d}{3R-d} \right) - b \right] - R^2$$

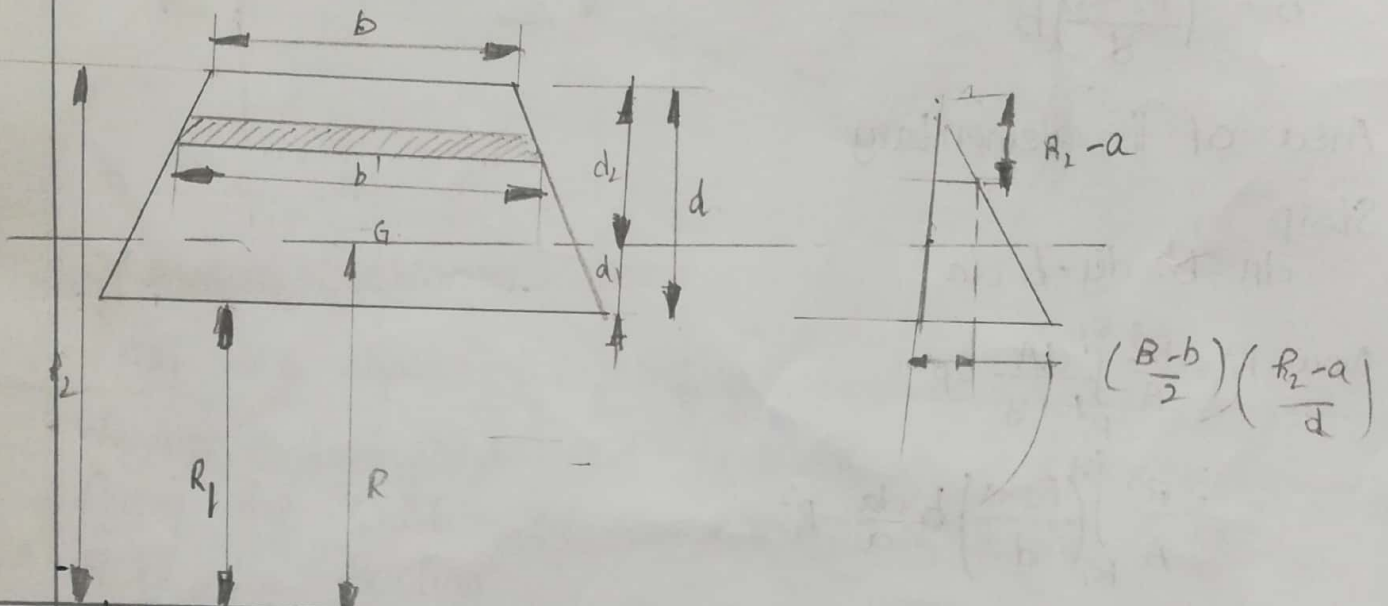
$$\text{or, } h^2 = \frac{2R^3}{d} \left[\left(\frac{3R+2d}{3d} \right) \log_e \left(\frac{3R+2d}{3R-d} \right) - 1 \right] - R^2$$



Triangular section.

Trapezoidal Section:-

Fig 20.5 shows a trapezoidal section. Consider an elemental strip of width b' and dy at distance y from the Centroidal axis



$$\text{Let, } R + y = a$$

$$dy = da$$

$$b' = b + \left(\frac{B-b}{d_1 + d_2} \right) (R_2 - a)$$

Area of the strip

$$dA = b' dy = \left[b + \left(\frac{B-b}{d_1 + d_2} \right) (R_2 - a) \right] da$$

$$I^2 = \frac{R^3}{A} \int_{R_1}^{R_2} \left[b + \left(\frac{B-b}{d_1 + d_2} \right) (R_2 - a) \right] \frac{da}{a} - R^2$$

$$= \frac{R^3}{A} \left[\int_{R_1}^{R_2} \frac{b da}{a} + \left(\frac{B-b}{d_1 + d_2} \right) \int_{R_1}^{R_2} \left(\frac{R_2 - a}{a} \right) da \right] - R^2$$

$$= \frac{R^3}{A} \left[b \cdot \log_e \frac{R_2}{R_1} + \left(\frac{B-b}{d_1 + d_2} \right) \left[R_2 \log_e a - a \right]_{R_1}^{R_2} \right] - R^2$$

$$\begin{aligned}
 &= \frac{R^3}{A} \left[b \cdot \log_e \frac{R_2}{R_1} + \left(\frac{B-b}{d_1+d_2} \right) \left\{ R_2 \log_e \frac{R_2}{R_1} - (R_2-R_1) \right\} \right] - R^2 \\
 &= \frac{R^3}{A} \left[b \cdot \log_e \left(\frac{R+d_2}{R-d_1} \right) + \left(\frac{B-b}{d_1+d_2} \right) (R+d_2) \log_e \left(\frac{R+d_2}{R-d_1} \right) - (B-b) \right] - R^2 \\
 &= \frac{R^3}{A} \left[b \cdot \log_e \left(\frac{R+d_2}{R-d_1} \right) + \left(\frac{B-b}{d} \right) (R+d_2) \log_e \left(\frac{R+d_2}{R-d_1} \right) - (B-b) \right] - R^2
 \end{aligned}$$

Where

$$A = \left(\frac{B+b}{2} \right) d$$

$$d_1 = \frac{d}{3} \left(\frac{B+2b}{B+b} \right)$$

$$d_2 = d - d_1$$

T-Section

Fig 20.6 shows a T-section

Let, $R+y=a$

$$dy = da$$

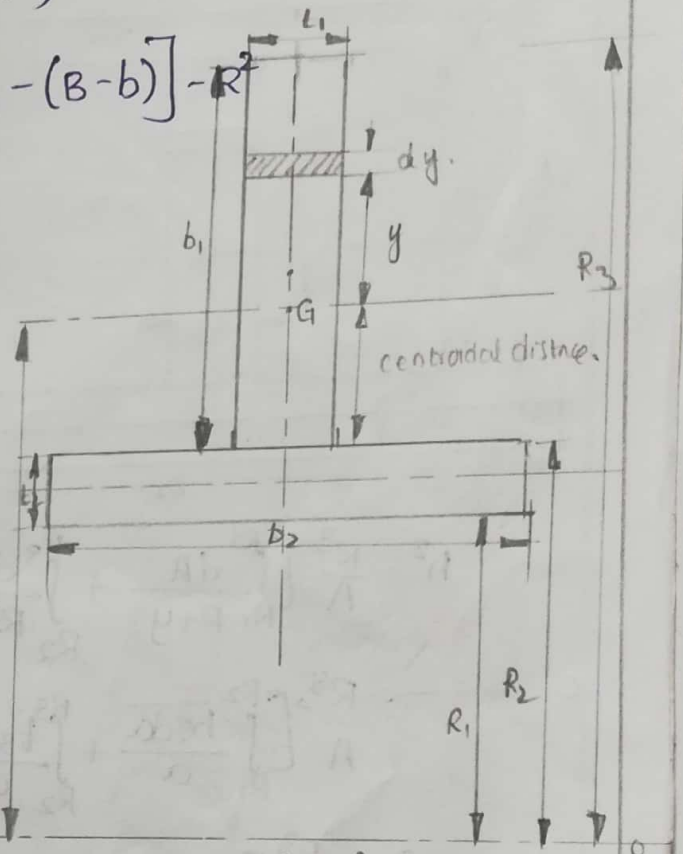
$$h^2 = \frac{R^3}{A} \left[\int_{R_1}^{R_2} \frac{dA}{R+y} + \int_{R_2}^{R_3} \frac{dA}{R+y} \right] - R^2$$

$$= \frac{R^3}{A} \left[\int_{R_1}^{R_2} \frac{b_2 da}{a} + \int_{R_2}^{R_3} \frac{t_1 da}{a} \right] - R^2$$

$$= \frac{R^3}{A} \left[b_2 \log_e \frac{R_2}{R_1} + t_1 \log_e \frac{R_3}{R_2} \right] - R^2$$

$$\text{i.e. } h^2 = \frac{R^3}{A} \left[b_2 \log_e \frac{R_2}{R_1} + t_1 \log_e \frac{R_3}{R_2} \right] - R^2$$

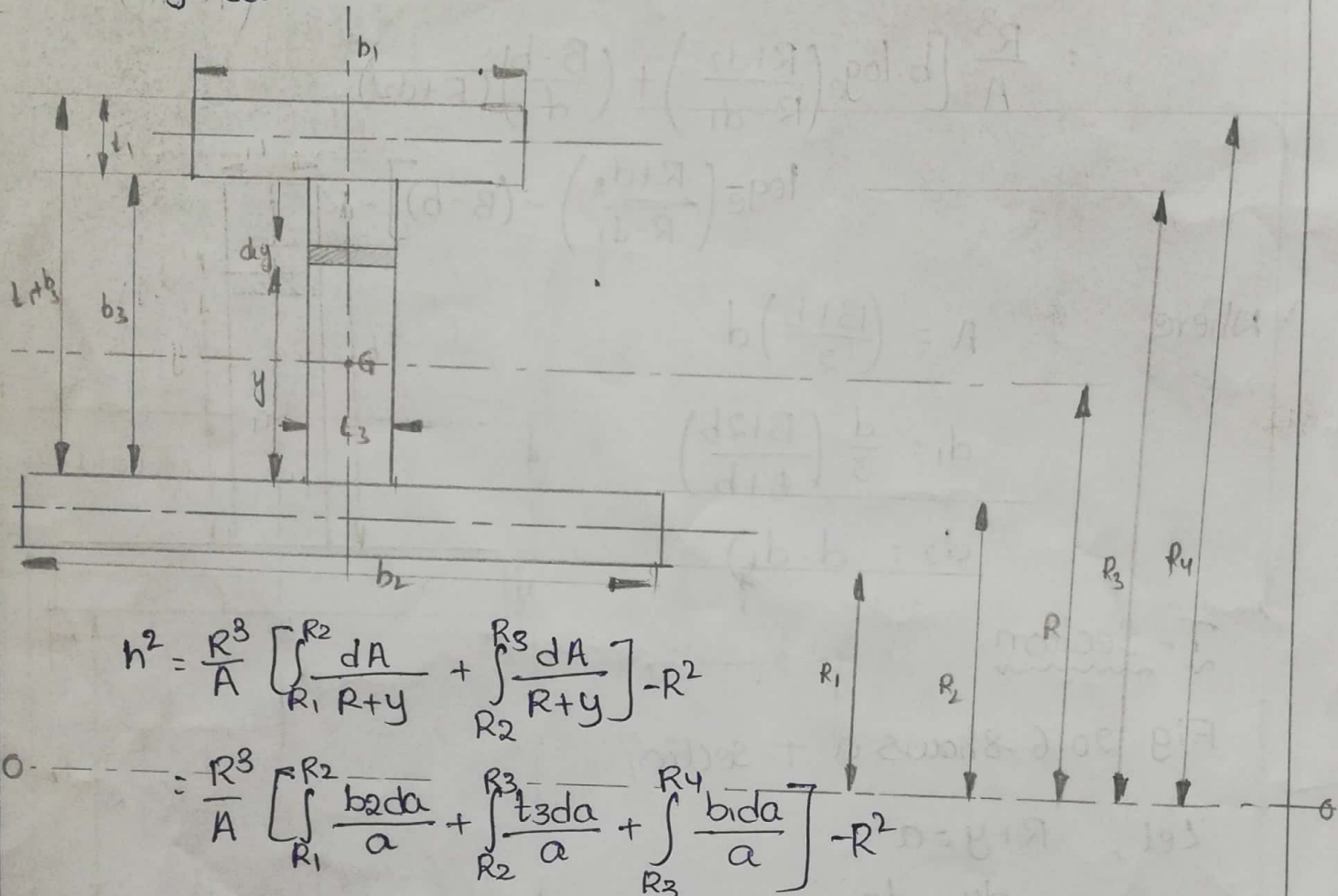
Where $A = b_1 t_1 + b_2 t_2$



1- Section

Consider the T-Section shown in Fig 20.7

Let, $R+y=a$
 $dy=da$



$$h^2 = \frac{R^3}{A} \left[\int_{R_1}^{R_2} \frac{dA}{R+y} + \int_{R_2}^{R_3} \frac{dA}{R+y} \right] - R^2$$

$$= \frac{R^3}{A} \left[\int_{R_1}^{R_2} \frac{b_2 da}{a} + \int_{R_2}^{R_3} \frac{t_3 da}{a} + \int_{R_3}^{R_4} \frac{b_1 da}{a} \right] - R^2$$

$$\text{Or } h^2 = \frac{R^3}{A} \left[b_2 \log_e \frac{R_2}{R_1} + t_3 \log_e \frac{R_3}{R_2} + b_1 \log_e \frac{R_4}{R_3} \right] - R^2$$

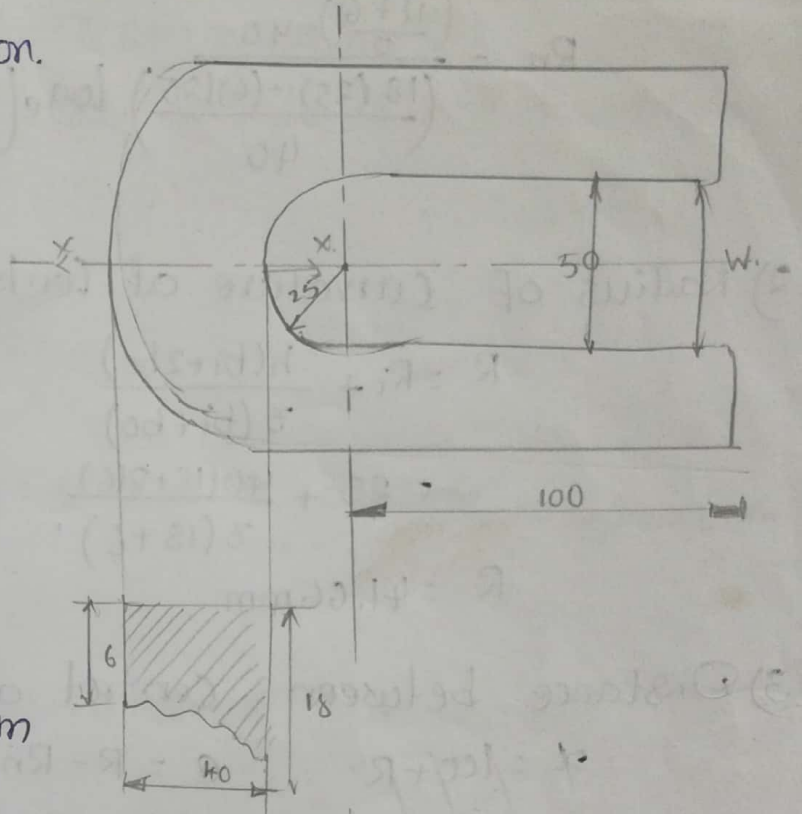
Where, $A = b_1 t_1 + b_2 t_2 + b_3 t_3$.

1) The frame of T punch stresses Find the stress in inner and out surface Net force of the frame $W=5000N$.

A) Given data:-

Given diagram is

trapezoidal cross section.



Top view of given problem

$$\text{Load } (W) = 5000N$$

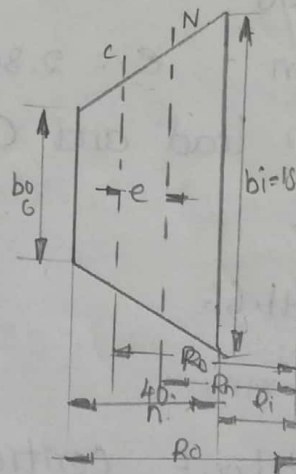
$$b_i = 18mm$$

$$b_o = 6mm$$

$$h = 40mm$$

$$R_i = 25mm$$

$$R_o = 25 + 40 = 65mm$$



From data book 4.21

Area of cross section at x-x

$$A = \left(\frac{b+d}{2} \right) h$$

$$= \left(\frac{18+6}{2} \right) (40)$$

$$A = 480mm^2$$

① Radius of curvature of Neutral axis

$$R_n = \left(\frac{b_i + b_o}{2} \right) h$$

$$\frac{(b_i R_o - b_o R_i)}{h} \log_e \left(\frac{R_o}{R_i} \right) - (b_i - b_o) \quad \left(\because \text{From 4.29, 9 data book} \right)$$

$$R_n = \frac{\left(\frac{11+6}{2} \right) 40}{\left(\frac{18(25) - (6)(25)}{40} \right) \log_e \left(\frac{65}{25} \right) - (18-6)} = 38.81 \text{ mm}$$

$$= 39 \text{ mm}$$

2) Radius of curvature at Centroidal axis.

$$R = R_i + \frac{h(b_i + 2b_o)}{3(b_i + b_o)}$$

$$= 25 + \frac{40(18 + 2(6))}{3(18 + 6)}$$

$$R = 41.66 \text{ mm}$$

3) Distance between Centroid and Neutral axis.

$$x = 100 + R$$

$$= 100 + 41.66$$

$$x = 141.66 \text{ mm}$$

$$e = R - R_n$$

$$= 41.67 - 38.81$$

$$e = 2.86 \text{ mm}$$

4) Distance between load and Centroidal axes.

$$x = 100 + R$$

$$= 100 + 41.66$$

$$x = 141.66 \text{ mm}$$

(5) Bending moment about centroidal axis

$$M = W \times x = 5000 \times 141.66 = 708.350 \text{ kN-mm}$$

The section x-x is subjected to of direction tensile load of 5000N and bending moment of 708.35 kN-mm

Therefore direct tensile stress is.

$$T_L = \frac{W}{A} = \frac{5000}{480} = 10.4 \text{ N/mm}^2$$

(6) Distance from neutral axis to inside of fibre

$$y_i = R_n - R_i = 38.81 - 25 = 13.81 \text{ mm}$$

(7) Distance from neutral axis to outside of fibre is

$$y_o = R_o - R_n = 65 - 38.81 = 26.19 \text{ mm}$$

(8) Maximum Bending stress at inside fibre for unsymmetric

$$\begin{aligned} \text{Section } T_{bi} &= \frac{M y_i}{A e R_i} \\ &= \frac{708.35 \times 10^3 (13.81)}{480 (2.86) (25)} \\ &= 285.03 \text{ N/mm}^2 \end{aligned}$$

(9) The maximum bending stress at outside unsymmetric

$$\begin{aligned} \text{Section } T_{bo} &= \frac{M y_o}{A e R_o} \\ &= \frac{708350 (26.19)}{480 (2.86) (65)} = 207.9 \text{ MPa} \end{aligned}$$

(10) Resultant stress at inner substance is

$$\begin{aligned} T &= T_L + T_{bi} \\ &= 10.4 + 285.83 = 303.2 \end{aligned}$$

(11) Resultant stress at outer surface

$$T = T_L - T_{bo}$$

$$T = 10.4 - 207.9 = -201.95 \text{ (-ve sign means compression)}$$

2) In a Crane hook carries a load of 20 kN as shown in figure the section at x-x is rectangular whose horizontal side is 100 mm. Find the stresses in inner and outer fibre at given section.

1) Given data

$$\text{Load (W)} = 20 \text{ kN} = 20 \times 10^3 \text{ N}$$

$$b = 20 \text{ mm}$$

$$h = 100 \text{ mm}$$

$$R_o = 150 \text{ mm}; R_i = R_o - h = 150 - 100 = 50 \text{ mm}$$

From data book (p. 21)

Area of Cross section at x-x

$$A = bh = 100 \times 20 = 2000 \text{ mm}^2$$

(1) Radius of Curvature at Neutral axis (R_n)

$$R_n = \frac{h}{\log_e \left(\frac{R_o}{R_i} \right)}$$

$$= \frac{100}{\log_e \left(\frac{150}{50} \right)}$$

$$R_n = 91.02 \text{ mm}$$

(2) Radius of Curvature of Centroidal axis

$$R = R_i + \frac{h}{2} = 50 + \frac{100}{2} = 100 \text{ mm}$$

(3) Distance b/w Centroidal axis and neutral axis

$$e = R - R_n$$

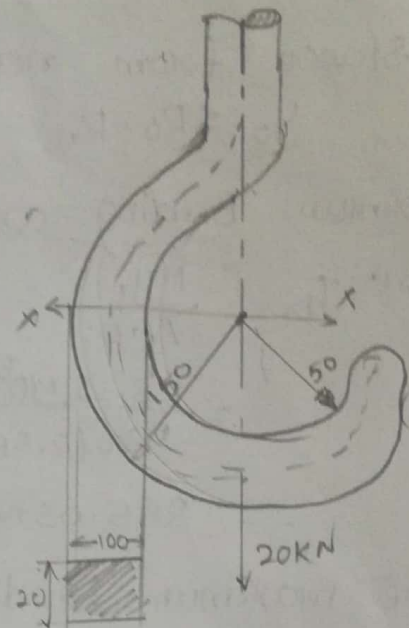
$$= 100 - 91$$

$$e = 9 \text{ mm}$$

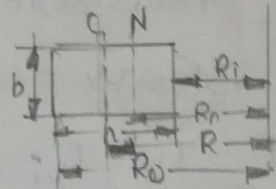
4) Distance b/w load and Centroidal axis

$$x = R = 100 \text{ mm}$$

5) Bending



All dimensions are in mm.



Bending Moment about Centroidal axis in

$$M = W \times x$$

$$= 2000 \times (100)$$

$$M = 200 \text{ k N.m}$$

6) The section at x-x is subjected to direct tensile stress when load of 20 kN at Area is

$$T_t = \frac{W}{A} = \frac{20 \times 10^3}{2000} = 10 \text{ MPa}$$

7) Distance b/w neutral axis to inside fibre

$$y_i = R_o - R_n = 150 - 91.02 = 59 \text{ mm}$$

8) Distance from neutral axis to outside fibre

$$y_o = R_o - R_n = 150 - 91.02 = 59 \text{ mm}$$

9) Maximum bending stress at inside be for unsymmetrical section.

$$T_{bi} = \frac{My_i}{AeR_i} = \frac{(20000)(100)(41.02)}{(2000)(9)(50)} = 22.7 \text{ MPa}$$

10) Maximum bending stress at outside fibre for Centroidal section

$$T_{bo} = \frac{My_o}{AeR_o} = \frac{(20000)(100)(59)}{(2000)(9)(150)} = 43.7 \text{ MPa}$$

11) Result stress at inside fibre for Centroidal section

$$\therefore T = T_t + T_{bi} = 10 + 22.7 = 32.7 \text{ MPa}$$

12) Resultant stress at outside fibre for Centroidal section.

$$\therefore T = T_t - T_{bo} = 10 - 43.7 = -33.7 \text{ MPa}$$

(-ve Sign indicates compressive stress)

3) A seaclamp is subjected to a max. load of W as shown in figure. The maximum tensile stress in the clamp is limited to 140 MPa . Find the value of load (W)

A) Given data:-

Maximum tensile stress $\sigma_{\text{tens}} = 140 \text{ MPa}$

From data book 4.21

Area of T section is

$$A = (3 \times 22) + (3 \times 19)$$

$$= 66 + 57$$

$$A = 123 \text{ mm}^2$$

(i) Radius of curvature at Neutral axis

$$R_o = 50 \text{ mm}; R_i = 25 \text{ mm}$$

$$(i) R_n = \frac{t_i(b_i - t) + t \cdot h}{(b_i - t) \log_e \left(\frac{R_i + t_i}{R_i} \right) + t \log_e \left(\frac{R_o}{R_i} \right)}$$

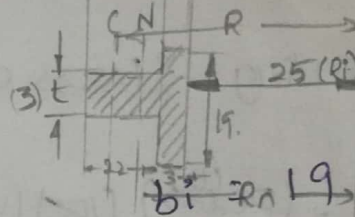
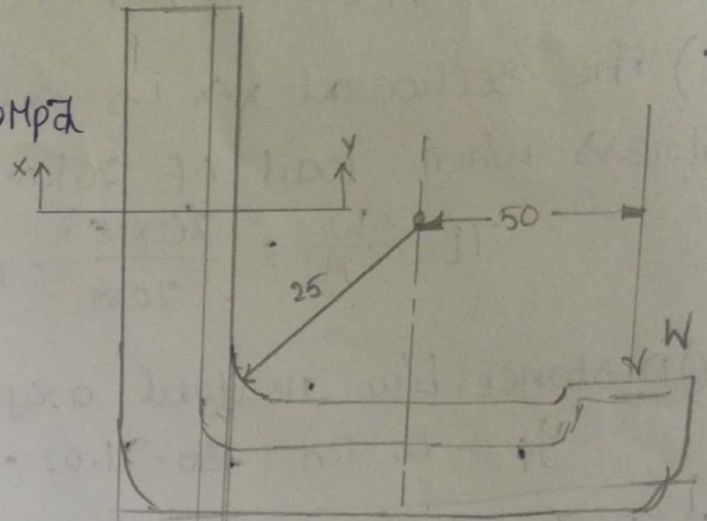
$$= \frac{3(19 - 3) + 3(25)}{(19 - 3) \log_e \left(\frac{25 + 3}{25} \right) + 3 \log_e \left(\frac{50}{25} \right)}$$

$$R_n = 31.59 \text{ mm}$$

$$(ii) R = R_i + \frac{\frac{1}{2} h^2 t + \frac{1}{2} t_i^2 (b_i - t)}{h t + t_i (b_i - t)}$$

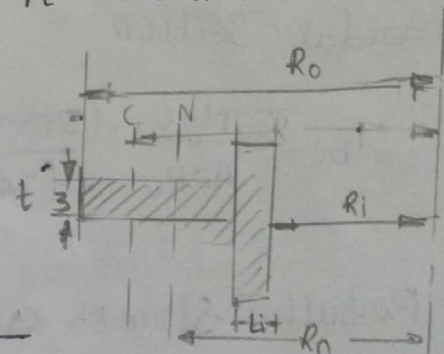
$$= 25 + \frac{\frac{1}{2} (25)^2 (3) + \frac{1}{2} (3)^2 (19 - 3)}{25(3) + 3(19 - 3)}$$

$$R = 33.20 \text{ mm}$$



$$\begin{aligned} t &= 3 \text{ mm} \\ t_i &= 3 \text{ mm} \\ h &= 25 \text{ mm} \end{aligned}$$

$$x = R_o + R$$



Distance b/w Centroidal axis and Neutral axis

$$e = R - R_n = 33.20 - 31.59 = \underline{2 \text{ mm}}$$

4) Distance b/w load and Centroidal axis is

$$x = 50$$

$$= 50 + 33.20 = 83.20 \text{ mm}$$

5) Bending moment about Centroidal axis is

$$M = W \times x$$

$$M = \underline{W \times (83.20) \text{ N mm}}$$

6) The section x-x is subjected to direct tension stress when load (W) at Area is

$$T_t = \frac{W}{A} = \frac{W}{123} \text{ N/mm}^2$$

7) Distance b/w Neutral axis & Inside of fibre is

$$y_i = R_n - R_i = 31.59 - 25 = 6.59 \text{ mm}$$

8) Distance b/w Neutral axis & outside of fibre

$$y_o = R_o - R_n = 50 - 31.59 = 18.41 \text{ mm}$$

9) Maximum bending stress at inside of fibre

$$T_{bi} = \frac{My_i}{AeR_i} = \frac{(6.59)}{(123)(2)(25)}$$

$$T_{bi} = 0.0089W$$

10) Resultant stress at inside fibre is

$$T = T_t + T_{bi}$$

$$= [0.00813 + 0.059]W$$

$$140 = 0.06(W)$$

$$\boxed{W = 2139 \text{ kN}}$$

* Gears *

①

* Introduction :-

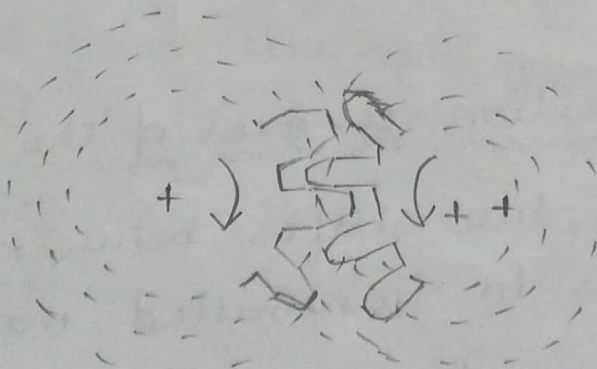
We have discussed earlier that the slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity is of importance (as in watch mechanism), the only positive drive is by "gears (or) toothed wheels". A gear drive is also provided, when the distance between the driver and the follower is very small.

* Classification of Gears :-

The gears (or) toothed wheels may be classified as follows :

* According to the type of gearing :- The gears, according to the type of gearing, may be classified as :

- a) External Gearing
- b) Internal Gearing
- and
- c) Rack and pinion



(a) External Gearing

In External gearing, the gears of two shafts mesh externally with each other as shown in fig. 28.4(a). The larger of these two wheels is called Spur wheel (or) gear and the similar wheel is called pinion. In an external gearing, the motion of two wheels is always unlike, as shown in the fig. 28.4(a).

In internal Gearing, the gears of two shafts mesh internally with each other as shown in the fig. 28.4(b). The larger of these two wheels is called annular wheel and the similar wheel is called pinion. In an internal gearing, the motion of the wheels is always like as shown in fig 28.4(b).

Sometimes, the gear of the shaft meshes externally and internally with the gears in a straight line as shown in fig 28.5. Such a type of gear is called rack and pinion. The straight line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of rack and pinion we can convert linear motion into rotary motion and vice-versa as shown in the fig 28.5.

* According to the position of axes of the shaft:-

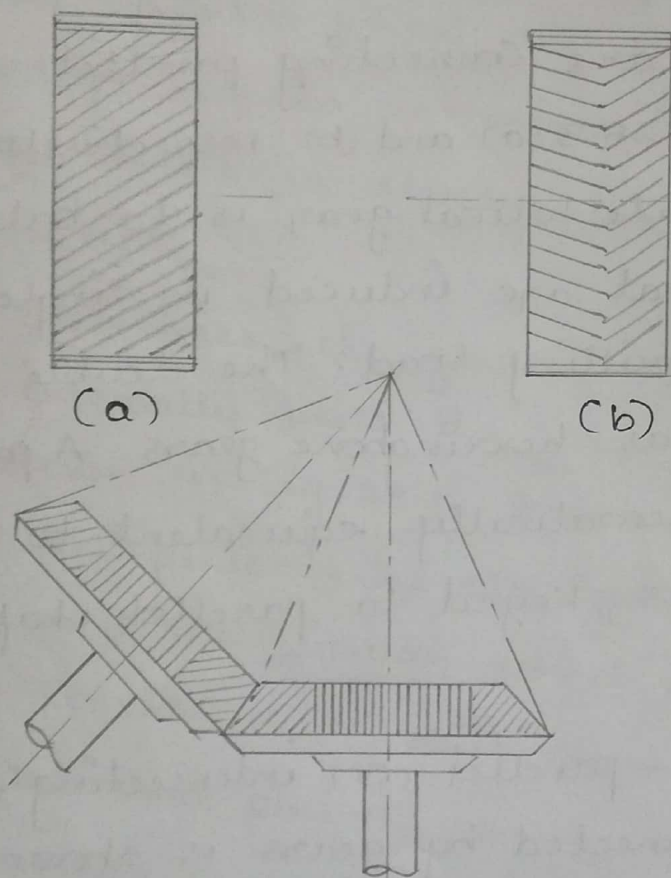
The axes of the two shafts between which the motion is to be transmitted, may be (a) parallel (b) Intersecting and

Non-intersecting & Non-parallel. (2)

The two parallel and Co-planar shafts connected by the gears as shown in the figure 28.2. These gears are called Spur gears and the arrangement is known as spur gearing. These gears have teeth parallel to the axis of the wheel as shown in the fig 28.2. Another name given to the spur gearing is helical gearing, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in fig 28.3 (a) and (b) respectively. The object of the double helical gear is to balance out the end thrusts that are induced in single helical gears which transmitting load. The double helical gears are known as herringbone gears. A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to parallel shaft have line contact.

The two non-parallel (or) intersecting, but coplanar shafts connected by gears is shown in fig 28.3 (c). The gears are called Bevel gears and the arrangement is known as bevel gearing. The bevel gears, like spur gears may also have their teeth inclined to the face of the bevel, in which case they are known as helical bevel gears.

The two non-intersecting and non-parallel i.e. non-coplanar shafts connected by gears is shown in fig. 28.3 (d). These gears are called skew bevel gears (or) spiral gears and the arrangement is known as skew bevel gearing (or) spiral gearing. This type of gearing also has a line contact, the rotation of which about the axes generates the two pitch surfaces known as hyperboloids.



(a)

(b)

- (a) Single helical Gear
- (b) Double Helical Gear.

Bevel Gear

* According to the peripheral velocity of gears :-

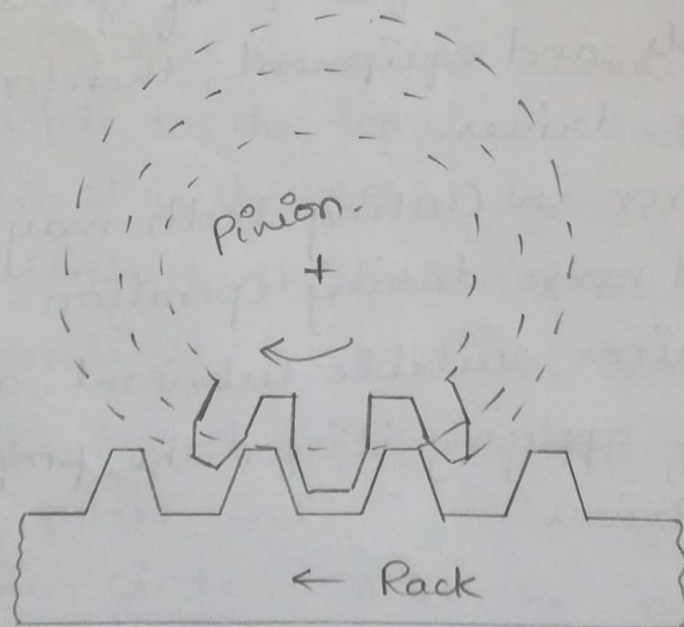
The gears, according to the peripheral velocity of the gears, may be classified as :-

- (a) Low velocity
- (b) Medium velocity and
- (c) High velocity.

The gears having velocity less than 3m/sec ^{(3) are} termed as low velocity gears and gears having velocity between 3 and 15m/sec known as medium velocity gears. If the velocity of gears is more than 15m/sec , then these are called high velocity gears.

* According to the position of teeth on the gear surface :- The teeth on the gear surface may be (a) Straight (b) Inclined (c) Curved

We have discussed earlier that the spur gears have straight teeth whereas helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.



* Advantages and disadvantages of Gear drive

The following are the advantages and disadvantages of the gear drive as Compared to other devices i.e., belt, rope and chain drives :-

Advantages :-

- 1) It transmits exact velocity ratio
- 2) It may used to transmit large power
- 3) It may be used for small Centre distances of shafts.
- 4) It has high efficiency.
- 5) It has reliable service.
- 6) It has Compact layout.

Disadvantages :-

- 1) Since the manufacture of gears require special tools and equipment, therefore it is Costlier than other drives.
- 2) The error in Cutting teeth may Cause vibrations and noise during operation
- 3) It requires suitable lubricant and reliable method of applying it, for the proper operation of gear drives.

* Terms used in Gears :- The following terms which will be mostly used in this Chapter.

- 1) pitch Circle :- It is an imaginary Circle which by

are rolling action, would give the same motion (4) as the actual gear.

- 2) pitch circle diameter :- It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also called as "pitch diameter".
- 3) pitch point :- It is a common point of contact between two surfaces.
- 4) pitch surface :- It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.
- 5) pressure angle (or) Angle of obliquity :- It is the angle between the common normal to two gear teeth at the point of contact and common tangent at the pitch point. It is denoted by " ϕ ". The standard pressure angles are $14\frac{1}{2}^\circ$ and 20° .
- 6) Addendum :- It is the radial distance of a tooth from the pitch circle to the top of the tooth.
- 7) Dedendum :- It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.
- 8) Addendum circle :- It is the circle drawn through the top of the teeth and is concentric with the pitch circle.
- 9) Dedendum circle :- It is the circle drawn through the bottom of the teeth. It is also called "Root circle".
- 10) Circular pitch :- It is the distance measured on

the circumference of the pitch circle from a point on one tooth to the corresponding point on the next tooth. It is usually denoted by " P_c ".

Mathematically

$$\text{Circular pitch } P_c = \pi D / T$$

D = Dia of the pitch circle

T = Number of teeth on the wheel

Note :- If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively then for them to mesh correctly.

$$P_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} = \frac{\pi D_3}{T_3} \quad (\text{or}) \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

Diametrical pitch :- It is the ratio of number of teeth to the pitch Circle diameter in millimeters. It is denoted by " P_d ". Mathematically,

$$P_d = \frac{T}{D} = \frac{\pi}{P_c}$$

12) Module :- It is the ratio of the pitch Circle diameter in millimeters to the number of teeth. It is usually denoted by " m ". Mathematically,

$$m = D/T$$

13) Clearance :- It is the radial distance from the top of the tooth to the bottom of the tooth in a meshing gear. A Circle passing through the top of the meshing gear is known as "Clearance Ratio".

14) Total depth :- It is the radial distance between the addendum and the dedendum Circle of a gear. It is equal to the sum of the addendum and dedendum.

15) Working depth :- It is the radial distance from the addendum Circle to the clearance Circle. It is equal to the sum of the addendum of two meshing gears.

16) Tooth Thickness :- It is the width of the tooth measured along the pitch Circle.

17) Tooth Space :- It is the width of the space between two adjacent teeth measured along the pitch Circle.

18) Backlash :- It is the difference between the tooth space and the tooth thickness, as measured on the pitch Circle.

19) Face of the tooth :- It is surface of the tooth above the pitch surface.

20) Top land :- It is the surface of top of the tooth.

- 21) Flank of the tooth :- It is the surface of tooth below the pitch surface.
- 22) Face width :- It is the width of the gear tooth measured parallel to its axis.
- 23) profile :- It is the curve formed by the face and flank of the tooth.
- 24) Fillet Radius :- It is the radius that connects the root circle to the profile of the teeth.
- 25) path of Contact :- It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.
- 26) Length of the path of Contact :- It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.
- 27) Arc of Contact :- It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts i.e.
- (a) Arc of approach :- It is the portion of the path of contact from the beginning of engagement to the pitch point.
- (b) Arc of recess :- It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Systems of Gear Teeth :-

6

The following four systems of gear teeth are commonly used in practice.

- 1) $14\frac{1}{2}^\circ$ Composite system
- 2) $14\frac{1}{2}^\circ$ Full depth involute system
- 3) 20° Full depth involute system
- 4) 20° Stub involute system.

The $14\frac{1}{2}^\circ$ Composite system is used for general purpose gears. It is stronger but has no interchangeability. The tooth profiles of the system has Cycloidal Curves at the top and bottom and involute Curve at middle portion. The teeth are produced by formed milling cutters (or) Hobs. The tooth profile of the $14\frac{1}{2}^\circ$ full depth involute system was developed for use with gear hobs for spur and helical gears.

The tooth profile of the 20° full depth involute system may be cut by hobs. The increase of the pressure angle from $14\frac{1}{2}^\circ$ to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base. The 20° stub involute system has a strong tooth to take heavy loads.

* Design Considerations for a Gear Drive :-

In this design for a gear drive, the following data is usually given :-

- 1) The power to be transmitted
- 2) The Speed of the driving gear
- 3) The Speed of the driven gear (or) the velocity ratio and
- 4) The Centre distance.

The following requirements must be met in the design of a gear drive :-

- (a) The gear drive should have sufficient strength so that they will not fail under static loading (or) dynamic loading during normal running conditions
- (b) The gear teeth should have wear characteristics so that their life is satisfactory
- (c) The use of space and material should be economical.
- (d) The alignment of the gears and deflections of the shafts must be considered because they effect on the performance of the gears.
- (e) The lubrication of the gears must be satisfactory.

* permissible working stress for gear teeth
in the Lewis Equation.

$$\sigma_w = \sigma_0 \times C_v$$

σ_0 = Allowable static stress and

C_v = velocity factor.

(7)

→ $C_v = \frac{3}{3 + v}$; for ordinary cut gears operating at velocities upto 12.5 m/sec.

→ $C_v = \frac{4.5}{4.5 + v}$; for Carefully cut gears operating at velocities upto 12.5 m/sec.

→ $C_v = \frac{6}{6 + v}$; for very accurately cut and ground metallic gears operating at velocities upto 20 m/sec.

→ $C_v = \frac{0.75}{0.75 + \sqrt{v}}$; for precision gears cut with high accuracy and operating at velocities upto 20 m/sec.

→ $C_v = \left(\frac{0.75}{1 + v} \right) + 0.25$; for Non-metallic gears.

In the above expressions, "v" is the pitch line velocity in (m/s).

* Design procedure for Spur Gears :-

i) First of all, the design tangential tooth load is obtained from the power transmitted & the pitch line velocity by using the following relation.

$$W_T = \frac{P}{v} \times C_S$$

W_T = permissible tangential tooth load (N)

P = power transmitted in (Watts)

$$v = \frac{\pi D N}{60} \quad \text{Here } D = \text{pitch circle diameter}$$

N = Speed in (RPM)

C_S = Service factor.

2. Apply the Lewis equation as follows :-

$$\begin{aligned} W_T &= \sigma_w \cdot b \cdot P_c \cdot y = \sigma_w \cdot b \cdot \pi \cdot m \cdot y \\ &= (\sigma_0 \cdot C_v) \cdot b \cdot \pi \cdot m \cdot y \end{aligned}$$

$$\left\{ \because C_v = \frac{\sigma_w}{\sigma_0} \right\}$$

Note:- The face width (b) may be taken as $3P_c$ to $4P_c$ (9.5m to 12.5m) for Cut teeth and $2P_c$ to $3P_c$ (6.5m to 9.5m) for Cast teeth.

3. Calculate the dynamic load (W_D or F_D) on the tooth by using "Buckingham Equation"

$$W_D = W_T + W_I$$

(or)

$$F_D = F_T + F_I$$

$$= F_T + \left(\frac{21v (b \cdot C + W_T)}{21v + \sqrt{b \cdot C + W_T}} \right)$$

$$F_T \text{ (or) } W_T = \frac{P}{v}$$

(8)

4. Find the static tooth load (beam strength) by using relation.

$$W_S = \sigma_c \cdot b \cdot P_c \cdot y = \sigma_c \cdot b \cdot \pi \cdot m \cdot y$$

$$(W_S > W_D)$$

5. Finally, find the wear tooth load by using the relation

$$W_W = D_p \cdot b \cdot Q \cdot k.$$

the wear load (W_W) should not be less than the dynamic load (W_D)

Helical Gears :-

(9)

* Introduction :- A helical gear has teeth in form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gears. The helices may be right handed on one gear and left handed on the other. The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contact, as in spur gearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with we have also a high efficiency of transmission.

* Terms used in Helical gears :-

The following terms in Condition with helical gears, as shown in fig 29.1 are important from the subject point of view.

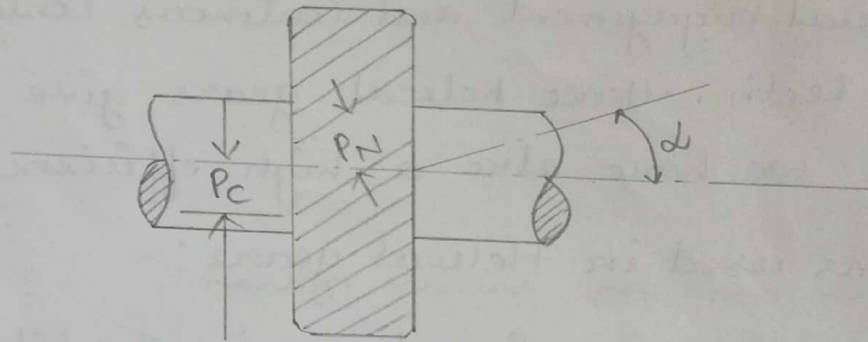
1. Helix angle :- It is the constant angle made by the helices with the axis of rotation.
2. Axial pitch :- It is the distance, parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by " P_c ". The axial pitch may also be defined as the circular pitch in the plane of rotation (or)

the diametral plane.

3. Normal pitch:- It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by " P_N ". The Normal pitch may also be defined as the Circular pitch in the normal plane which is a plane perpendicular to the teeth.

Mathematically, a normal pitch;

$$P_N = P_C \cos \alpha$$



* Helical gear Nomenclature *

* Formative (or) Equivalent number of teeth for helical gears:- The formative (or) Equivalent number of teeth for a helical gear may be defined as the number of teeth can be generated on the surface of a cylinder having a radius equal to the radius of curvature at the point at the tip of the minor axis of an ellipse obtained by taking a section of the gear in the normal plane

Mathematically, formative (or) Equivalent number of teeth on a helical gear

$$T_E = T / \cos^3 \alpha$$

T = Actual Number of teeth on helical gear
and

β (or) α = Helix angle.

* Strength of Helical gears :- In helical gears the Contact between mating teeth is gradual starting at one end and moving along the teeth so that at any instant the line of Contact runs diagonally across the teeth. Therefore in order to find the strength of helical gears, a modified Lewis equation is used. It is given by

$$F_b = [\sigma_b] \cdot b \cdot \pi \cdot m \cdot y_v$$

As per Data book

As per the text book :-

$$W_T = (\sigma_0 \times C_v) \cdot b \cdot m \cdot \pi \cdot y'$$

W_T = tangential load

σ_0 = Allowable static stress

C_v = velocity factor

b = Face width

m = module

y' = Tooth form factor.

*, The value of velocity factor C_v upon the velocities

$$C_v = \frac{6}{6 + v} ; \text{ for peripheral velocities from } 5 \text{ m/sec to } 10 \text{ m/sec.}$$

$$C_v = \frac{15}{15 + v} ; \text{ for } (10 \text{ m/sec to } 20 \text{ m/sec})$$

$$C_v = \frac{0.75}{0.75 + \sqrt{v}} ; \text{ for } (> 20 \text{ m/sec})$$

$$C_v = \frac{0.75}{1 + v} + 0.25 ; \text{ for Non-metallic gears}$$

*, The dynamic tooth load on the helical gear is given by

$$W_D = W_T + \frac{21v(b \cdot C \cos^2 \alpha + W_T) \cos \alpha}{21v + \sqrt{b \cdot C \cos^2 \alpha + W_T}}$$

$$\{ W_D = F_D ; W_T = F_T \}$$

$$*, W_T = F_T = \frac{P}{v}$$

*, The static tooth load (or) endurance strength of the tooth is given by

$$W_s = \sigma_e \cdot b \cdot \pi \cdot m \cdot y'$$

*, wear strength of gear tooth

$$W_w = \frac{D_p \cdot b \cdot Q \cdot k}{\cos^2 \alpha}$$

$$\{ \alpha = \beta = \text{Helix angle} \}$$

* Q = Ratio factor

(11)

$$\rightarrow Q = \frac{2i}{i+1} \quad (\text{Or}) \quad \frac{2z_2}{z_2+z_1} \quad \text{for External gears}$$

$$\rightarrow Q = \frac{2i}{i-1} \quad (\text{Or}) \quad \frac{2z_2}{z_2-z_1} \quad \text{for internal gears.}$$

$$*, K = \frac{\sigma_e^2 \sin^2 \alpha}{1.4} \left(\frac{1}{E_1} + \frac{1}{E_2} \right)$$

Gears (Spur & helical)

Design a spur gear drive to transmit 22 kW at 1000 rpm speed reduction is 2.5 the center distance b/w the gear shaft is approximately 350 mm the material of pinion C45 steel, gear wheel C2 grade 30 design the drive (use Lewis & buckingham equation)

3d Given data

$$\text{Power } P = 22 \text{ kW} = 22 \times 10^3 \text{ W}$$

$$\text{pinion speed } N_1 = 1000 \text{ rpm}$$

$$\text{velocity ratio } (i) = 2.5$$

$$\text{center distance } (a) = 350 \text{ mm}$$

$$\text{material for pinion} = \text{C 45 steel}$$

$$\text{material for gear wheel} = \text{C 2 grade 30}$$

$$\rightarrow \text{static load } F_t = P/v$$

$$\begin{aligned} v &= \frac{\pi d_1 n_1}{60} \\ &= \frac{\pi \times d_1 \times 1000}{60} \end{aligned}$$

$$a = \frac{d_1 + d_2}{2}$$

$$2a = d_1 + d_2$$

$$2(350) = d_1 + d_2$$

$$d_1 + d_2 = 700$$

we know that

$$i = d_2/d_1$$

$$d_2 = i d_1$$

$$d_2 = 2.5 d_1$$

$$d_1 + d_2 = 700$$

$$d_1 + 2.5d_1 = 700$$

$$3.5d_1 = 700$$

$$d_1 = 200 \text{ mm}$$

$$d_2 = 500 \text{ mm}$$

$$V = \frac{\pi \times 200 \times 1000}{60} = 10.471 \text{ m/s}$$

$$F_t = P/V$$

$$= 22 \times 10^3 / 10.471 = 2101.04 \text{ N.}$$

→ Design tooth load

$$F_D = F_t \times k_s \times C_v$$

$$k_s = 1.00 \quad (\text{from table 28.34})$$

$$C_v = \frac{6+V}{6} \quad : (V \leq 20)$$

$$= \frac{6+10.471}{6} = 2.745$$

$$F_D = 2101.04 \times 1.00 \times 2.745$$

$$= 5767.35 \text{ N}$$

→ face width & module

$$F_b = (\sigma_b) m \cdot b \cdot \pi \cdot y$$

$$(\sigma_b)_q = 60 \text{ N/mm}^2$$

$$(\sigma_b)_p = 140 \text{ N/mm}^2$$

(b) face width = 10 m

(assume initially 10)

2

$$z_1 = 20^\circ \text{ (teeth on pinion)}$$

$$z_2 = i z_1 \\ = 2.5 \times 20 = 50^\circ$$

for pinion

$$y = 0.154 - \frac{0.912}{z_1} \\ = 0.154 - \frac{0.912}{20} = 0.108.$$

For gear

$$y = 0.154 - \frac{0.912}{z_2} \\ = 0.154 - \frac{0.912}{50} = 0.135$$

Hence the strength factor for pinion

$$\text{for pinion} : \sigma_b \times y_p = 140 \times 0.108 \\ = 15.12 \text{ N/mm}^2$$

$$\text{for gear} : \sigma_b \times y_g = 60 \times 0.135 \\ = 8.1 \text{ N/mm}^2$$

$$F_b = \{(\sigma_b) \cdot m \cdot b \cdot \pi \cdot y\}_g \geq F_d$$

$$5767.35 = 60 \times 10 \text{ m} \times m \times \pi \times 0.135$$

$$m^2 = 22.66$$

$$m = 4.76 \approx 5 \text{ mm}$$

→ The corrected beam strength

$$\begin{aligned}(F_b)_g &= (F_b)_g \times 10m \times m \times \pi \times y \\ &= 60 \times 10(5) \times 5 \times \pi \times 0.135 \\ &= 6361.72 \text{ N}\end{aligned}$$

The above strength must be checked with max dynamic load

$$\rightarrow F_d = F_t + F_i$$

$$F_i = \frac{21v (cb + Ft)}{21v + \sqrt{cb + Ft}}$$

$$C = \frac{K \cdot E_1 E_2 \cdot e}{(E_1 + E_2)}$$

(for table 28.17) e_1 & e_2
(for table 28.35) e

$$C = \frac{0.111 \times 2.15 \times 10^5 \times 1.1 \times 10^5 \times 0.056}{(2.15 \times 10^5 + 1.1 \times 10^5)}$$

$$C = 452.313$$

$$\begin{aligned}F_i &= \frac{21 \times 10.5 \times (452.313 \times 10(5) + 2101.04)}{21 \times 10.5 + (\sqrt{452.313 \times 10(5) + 2101.04})} \\ &= 14430.3 \text{ N}\end{aligned}$$

$$\begin{aligned}F_d &= 2101.04 + 14430.3 \\ &= 16531.34 \text{ N}\end{aligned}$$

for safe design :-

$$(F_b)_g \geq F_d$$

$$m = 6 ; b = 10m \\ = 60$$

$$z_1 = d_1/m$$

$$= 200/6 = 34$$

$$z_2 = i z_1$$

$$= 2.5 \times 34 = 85$$

$$y = 0.154 - \frac{0.912}{85} = 0.143$$

$$\text{assume } (\sigma_b) = 70$$

$$(F_b)_g = 70 \times \pi \times 60 \times 6 \times 0.143$$

$$= 11321.04 \text{ N}$$

$$c = \frac{k \cdot \epsilon_1 \cdot \epsilon_2 \cdot e}{\epsilon_1 + \epsilon_2}$$

$$= \frac{0.111 \times 2.15 \times 10^5 \times 1.1 \times 10^5 \times 0.025}{(2.15 \times 10^5 + 1.1 \times 10^5)}$$

$$= 201.21$$

$$F_i = \frac{21 \times 10.5 \times (201.21 \times 60 + 2101.04)}{21 \times 10.5 + (\sqrt{201.21 \times 60 + 2101.04})}$$

$$= 9204.20$$

$$F_d = F_t + F_i$$

$$= 2101.04 + 9204.20$$

$$= 11305.24 \text{ N}$$

$$\text{Hence the } (F_b)_g \geq F_d$$

$$11321.04 \geq 11305.24$$

design is safe

→ checking for wear strength

$$F_w = Q \cdot b \cdot d_1 \cdot K_w$$

$$b = 60$$

$$Q = \frac{2i}{i+1} = \frac{2 \times 2.5}{2.5 + 1} = 1.42$$

$$d_1 = 200 \text{ mm}$$

$$K_w = 1.31 \quad (\text{from table 28.36})$$

$$\begin{aligned} F_w &= 1.42 \times 60 \times 200 \times 1.31 \\ &= 22322.4 \end{aligned}$$

$$F_w \geq F_d$$

$$22322.4 \geq 11305.24$$

design is safe

→ Other parameters for gear drive

$$\text{Addendum} \rightarrow 1m = 1 \times 6 = 6$$

$$\text{dedendum} \rightarrow 1.25m$$

$$= 1.25 \times 6 = 7.5$$

$$\begin{aligned} \text{tip circle dia for pinion} &\rightarrow d_1 + 2h_a \\ &= 200 + 2(6) = 212 \end{aligned}$$

$$\begin{aligned} \text{for gear} &\rightarrow d_2 + 2h_a \\ &= 500 + 2(6) = 512 \end{aligned}$$

$$\begin{aligned} \text{root circle dia for pinion} &\rightarrow d_1 + 2h_f \\ &= 200 + 2(7.5) = 185 \end{aligned}$$

$$\text{for gear} \rightarrow d_2 + 2h_f = 500 + 2(7.5) = 485$$

gear drive is required to transmit a max power of 22.5 kW. The velocity ratio is 1:2 and rpm of the pinion is 200. The approximately center distance b/w the shaft may be taken as 600 mm. The teeth has 20 stub involute profiles. The static stress for the gear materials which is C.D may be taken as 60 mpa and face width as 10 times the module. find the module, face width, & no. of teeth on each gear. check the design for dynamic & wear loads. the deformation or dynamic factor in the buckingham equation may be taken as 80 and the materials combination factor for the wear 1.4

sol Given data :

$$\text{Power } P = 22.5 \text{ kW} \\ = 22.5 \times 10^3 \text{ W}$$

$$\text{Pinion speed} = 200 \text{ rpm}$$

$$\text{velocity ratio} = 1:2$$

$$(\sigma_b)_g = (\sigma_b)_{pin} = 60 \text{ mpa}$$

$$\text{center distance } a = 600 \text{ mm}$$

$$\text{stub involute profile} = 20^\circ$$

$$\text{dynamic load } c = 80$$

$$k_w = 1.4$$

$$\text{face width } b = 10 m$$

$$i = d_2/d_1 = 2$$

$$a = \frac{d_1 + d_2}{2}$$

$$2a = d_1 + d_2$$

$$2(600) = d_1 + d_2$$

$$d_1 + d_2 = 1200$$

$$d_1 + 2d_1 = 1200$$

$$d_1 = 400 \text{ mm}$$

$$d_2 = 800 \text{ mm}$$

→ No. of teeth on pinion

$$z_2 = d_1/m$$

$$z_1 = 400/m$$

$$y = 0.175 - \frac{0.841}{z_1}$$

$$= 0.175 - \frac{0.841 \text{ m}}{400}$$

→ Lewis equation

$$(F_b)_p = \sigma_b \cdot b \cdot \pi \cdot m \cdot y$$

$$= 60 \times 10 \text{ m} \times \pi \times m \times (0.175 - 2.035 \times 10^{-3} \text{ m})$$

$$= 1884 \text{ m}^2 (0.175 - 2.035 \times 10^{-3} \text{ m})$$

$$= 329.7 \text{ m}^2 - 3.9611 \text{ m}^3$$

→ design tangential force

$$F_D = \frac{P \times k_s \times C_v}{v}$$

$$v = \frac{\pi d_1 n_1}{60}$$

$$= \frac{\pi \times 400 \times 200}{60} = 4.18 \text{ m/s}$$

$$C_v = \frac{3 + v}{3}$$

$$= \frac{3 + 4.18}{3} = 2.393$$

$$F_D = \frac{22.5 \times 10^3 \times 1.00 \times 2.393}{4.18}$$

$$= 12880.98 \text{ N}$$

$$\text{Now } F_D = (F_b)_p$$

$$12880.98 = 329.7 \text{ m}^2 - 3.9611 \text{ m}^3$$

$$3.9611 \text{ m}^3 - 329.7 \text{ m}^2 + 12880.98 = 0$$

$$m = 6.510 \quad (\text{from table 28.20})$$

$$m = 8$$

$$\text{face width } b = 10 \text{ m}$$

$$= 10 \times 8 = 80 \text{ mm}$$

→ No. of teeth on pinion

$$z_1 = d_1/m$$

$$= 400/8 = 50$$

$$i = z_2/z_1 = 2$$

$$z_2 = 2 \times 50 = 100$$

→ Lewis factor for pinion

$$y_p = 0.175 - \frac{0.841}{z_1}$$

$$= 0.175 - \frac{0.841}{50} = 0.158$$

$$(F_b)_p = (F_b)_p \cdot b \cdot \pi \cdot m \cdot y$$

$$= 60 \times 80 \times \pi \times 8 \times 0.158$$

$$= 19060.670 \text{ N}$$

→ max dynamic load

$$F_D = F_t + F_i$$

$$F_t = P/v$$

$$= 22.5 \times 10^3 / 4.18$$

$$= 5382.77 \text{ N}$$

$$F_i = \frac{21v (cb + F_t)}{21v + \sqrt{cb + F_t}}$$

$$= \frac{21 \times 4.18 (80 \times 80 + 5382.77)}{21 \times 4.18 + \sqrt{80 \times 80 + 5382.77}}$$

$$= 5268.168 \text{ N}$$

$$F_D = 5382.77 + 5268.168$$

$$= 10650.939 \text{ N}$$

$$\text{SO } (F_b)_p \geq F_D$$

$$19060.670 \geq 10650.93$$

design is safe

→ wear strength

$$F_w = d_1 \cdot b \cdot Q \cdot k_w$$

$$= 400 \times 80 \times Q \times 1.4$$

$$Q = \frac{2i}{i+1}$$

$$= \frac{2 \times 2}{2+1} = 1.33$$

$$F_w = 400 \times 80 \times 1.33 \times 1.4 = 59584.3 \text{ N}$$

Since both $(F_b)_p$ & F_w values are greater than the F_D

$$\therefore (F_b)_p \text{ \& } F_w \geq F_D$$

$$19060.67 \text{ \& } 59584.3 \geq 10650.93$$

design is safe

→ other parameters

$$\text{Addendum} \rightarrow 1m \rightarrow 1 \times 8 = 8 \text{ mm}$$

$$\text{dedendum} \rightarrow 1.25m \rightarrow 1.25 \times 8 = 10 \text{ mm}$$

$$\begin{aligned} \text{tip circle dia for pinion} &\rightarrow d_1 + 2ha \\ &= 400 + 2(8) = 416 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{root circle dia for pinion} &\rightarrow d_1 + 2hf \\ &= 400 + 2(10) = 380 \text{ mm} \end{aligned}$$

3. A bronze spur gear pinion rotating at 600 rpm drives a C-2 spur gear at a transmission ratio of 4. The allowable static stress for the bronze pinion and C-2 gear are 85 mpa & 105 mpa respectively. The pinion has 16 standard 20° full depth involute teeth of module 8 mm the face width of both gears is 90 mm. Find the power that can be transmitted from the stand point of strength

Sol Given data :

$$\text{Speed on pinion } (n_1) = 600 \text{ rpm}$$

$$\text{teeth on pinion} = 16$$

$$\text{module } m = 8 \text{ mm}$$

$$\text{face width } b = 90 \text{ mm}$$

$$\text{transmission ratio } i = Z_3/Z_1 = 4$$

$$(\sigma_b)_p = 84 \text{ mpa}$$

$$(\sigma_b)_g = 105 \text{ mpa}$$

$$\begin{aligned} \rightarrow (y)_p &= 0.154 - \frac{0.912}{Z_1} \\ &= 0.154 - \frac{0.912}{16} = 0.097 \end{aligned}$$

$$\begin{aligned} (y)_g &= 0.154 - \frac{0.912}{Z_2} \\ &= 0.154 - \frac{0.912}{4 \times 16} = 0.139 \end{aligned}$$

$$\rightarrow (\sigma_b)_p \times (y)_p = 84 \times 0.097 = 8.148$$

$$(\sigma_b)_g \times (y)_g = 105 \times 0.139 = 14.595$$

$$\begin{aligned} \rightarrow (F_b)_g &= (\sigma_b)_g \times b \times \pi \times m \times y \\ &= 105 \times 8 \times \pi \times 90 \times 0.139 \\ &= 33013.112 \text{ N} \end{aligned}$$

$$\rightarrow F_D = \frac{P \times K_B \times C_v}{v}$$

$$\begin{aligned} v &= \frac{\pi d_1 n}{60} \\ &= \frac{\pi \times 128 \times 600}{60} \\ &= 4.021 \text{ m/s} \end{aligned}$$

$$\begin{aligned} d_1 &= m Z_1 \\ &= 8 \times 16 = 128 \end{aligned}$$

$$\begin{aligned} C_v &= \frac{3+v}{3} \\ &= \frac{3+4.021}{3} \\ &= 2.34 \end{aligned}$$

$$FD = \frac{P \times k_s \times C_v}{V}$$

$$33013.112 = \frac{P \times 1 \times 2.34}{4.021}$$

$$P = 56728.94 \text{ W}$$

4. The following particulars of a single reduction of spur gear of given gear ratio 10, distance b/w centers 660 mm, pinion transmits 500 kW at 1800 rpm. Involute teeth of standard proportions with pressure angle of 22.5° . Find the nearest standard module if no interference is to occur. The no. of teeth on each wheel

Ad Given data

$$\text{gear ratio } i = 10$$

$$\begin{aligned} \text{Power } P &= 500 \text{ kW} \\ &= 500 \times 10^3 \text{ W} \end{aligned}$$

$$N = 1800 \text{ rpm}$$

$$\text{center distance } a = 660 \text{ mm}$$

$$\alpha = 22.5^\circ$$

→ No. of teeth required for pinion in order to avoid interference

$$Z_1 = \frac{2 A_w}{i \left[\left\{ 1 + \frac{1}{i} \left(\frac{1}{i} + 2 \right) \sin^2 \alpha \right\}^{1/2} - 1 \right]}$$

$$= \frac{2 \times 1}{10 \left[\left\{ 1 + \frac{1}{10} \left(\frac{1}{10} + 2 \right) \sin^2 22.5^\circ \right\}^{1/2} - 1 \right]}$$

$$= 13.105 \approx 14$$

$$\rightarrow a = \frac{d_1 + d_2}{2}$$

$$2a = d_1 + d_2$$

$$i = d_2/d_1 = 10$$

$$d_1 + 10d_1 = 2(660)$$

$$d_1 = 120 \text{ mm}$$

$$d_2 = 1200 \text{ mm}$$

→ module

$$m = d_1/z_1$$

$$= 120/14 = 8.57 \approx 8$$

$$\rightarrow z_1 = d_1/m$$

$$= 120/8 = \underline{15}$$

$$z_2 = d_2/m$$

$$= 1200/8 = \underline{150}$$

5. A pair of straight teeth spur gear having 20° involute full depth teeth is to transmit 12 kW at 300 rpm of the pinion, speed ratio is 3. The allowable static stress for gear & pinion of steel are 60 MPa & 105 MPa respectively. Assume the following module 6 mm, no. of teeth on pinion 16, face width 14 times the module, velocity factor $c_v = 4.5/4.5 + v$. Determine the pitch diameter of the gear, check the wear for gear given endurance strength 600 MPa, young's modulus of pinion 200 kN/mm² & young's modulus of gear is 100 kN/mm².

sol

Given data

$$\text{power } P = 12 \text{ kW}$$

$$N = 300 \text{ rpm}$$

$$\alpha = 20^\circ$$

$$\text{Speed ratio } i = 3$$

$$(\sigma_b)_p = 105 \text{ mpa}$$

$$(\sigma_b)_g = 60 \text{ mpa}$$

$$m = 6 \text{ mm}$$

$$z_1 = 16$$

$$b = 14 \text{ m}$$

$$C_v = \frac{4.5}{4.5 + v}$$

$$\sigma_e = 600 \text{ mpa}$$

$$e_1 = 200 \times 10^3 \text{ N/mm}^2$$

$$e_2 = 100 \times 10^3 \text{ N/mm}^2$$

$$\rightarrow m = d_1 / z_1$$

$$d_1 = m z_1$$

$$= 6 \times 16 = 96 \text{ mm}$$

$\rightarrow$ wear strength gear teeth

$$F_w = d_1 \cdot b \cdot Q \cdot k_w$$

$$Q = \frac{2i}{i+1} = \frac{2 \times 3}{3+1} = 1.5$$

$$k_w = \frac{\sigma_e^2 \sin \alpha}{1.4} \left(\frac{1}{e_1} + \frac{1}{e_2} \right)$$

$$= \frac{600^2 \sin 20}{1.4} \left(\frac{1}{200} + \frac{1}{100} \right)$$

$$= 87978.036 (0.015)$$

$$= 1.31922$$

$$b = 14 \times 6$$

$$= 84 \text{ mm}$$

$$F_w = 96 \times 84 \times 1.5 \times 1.31922$$

$$= 15957.291 \text{ N}$$

→ maximum dynamic load

$$F_d = F_t + F_i$$

$$(v = \pi d n / 60)$$

$$F_t = P/v$$

$$= 12 \times 10^3 / 1.5071$$

$$= 7955.7 \text{ N}$$

$$F_i = \frac{21v (cb + F_t)}{21v + \sqrt{cb + F_t}}$$

$$C = \frac{K \cdot E_1 \cdot E_2 \cdot e}{E_1 + E_2}$$

$$= \frac{0.111 \times (200 \times 10^3) \times (100 \times 10^3) \times 0.030}{(200 \times 10^3) + (100 \times 10^3)}$$

$$= 222$$

$$F_i = \frac{21 \times 1.5079 (222 \times 84 + 7957.7)}{21 \times 1.5079 + \sqrt{(222 \times 84 + 7957.7)}}$$

$$= 4330.415 \text{ N}$$

$$F_d = 7957.7 + 4330.415$$

$$= 12288.115 \text{ N}$$

$$F_w \geq F_d$$

$$15957.29 \geq 12288.115$$

design is safe

9
Pair of helical gear are to be transmit 15 kw the teeth are 20 stub mean dia plan and have a helix angle of 45° thick pinion runs at 10,000 rpm and as 80 mm pitch dia the gear as 320 mm pitch diameter if the gears are made of cast steel having allowable static strength of 100 mpa. determine a suitable module and face width from static strength considerations and check the gears for wear even given surface endurance strength 618 mpa

3d Given data

$$\text{Power } P = 15 \text{ kW}$$

Teeth are 20 stub

$$\text{helix angle } (\beta) = 45^\circ$$

$$N = 10,000$$

$$\text{Pitch dia } d_p = 80 \text{ mm}$$

$$\text{Pitch dia for gear } (d_g) = 320 \text{ mm}$$

$$\text{allowable static } (\sigma_b) = 100 \text{ mpa}$$

$$\text{endurance strength } (\sigma_e) = 618 \text{ mpa}$$

$$\rightarrow F_D = \frac{P \times K_s \times C_v}{v}$$

$$v = \frac{\pi d n}{60}$$

$$= \frac{\pi \times 80 \times 10,000}{60} = 41.887 \text{ m/s}$$

$$C_v = \frac{5.5 + \sqrt{v}}{5.5}$$

$$= \frac{5.5 + \sqrt{41.887}}{5.5} = 2.176$$

$$F_D = \frac{15 \times 10^3 \times 1.00 \times 2.176}{41.887}$$

$$= 779.6 \text{ N}$$

→ virtual number

$$z_v = \frac{z}{\cos^3 \beta}$$

$$z_1 = d_1/m$$

$$= \frac{d_1/m}{\cos^3 \beta}$$

$$= \frac{80/m}{\cos^3 45} = \frac{228.57}{m}$$

$$\rightarrow F_b = (f_b) \cdot b \cdot \pi \cdot m \cdot y_v$$

$$779.50 = 100 \times 10m \times \pi \times m \times y_v$$

$$y_v = \frac{0.175 - 0.841}{z}$$

$$= \frac{0.175 - 0.841}{228.57/m}$$

$$= 0.175 - 3.679 \times 10^{-3} m$$

$$779.50 = 100 \times 10m \times \pi \times m \times (0.175 - 3.679 \times 10^{-3} m)$$

$$779.50 = 3141.59 m^2 \times (0.175 - 3.679 \times 10^{-3} m)$$

$$779.50 = 594.77 m^2 - 11.55 m^3$$

$$11.55 m^3 - 594.77 m^2 + 779.50 = 0$$

$$m = 1.15 \text{ (for preferred \& assume module 3)}$$

$$m = 3$$

$$(f_b) 779.50 = 100 \times (10 \times 3) \times \pi \times 3 \times 0.163$$

$$f_b = 4635.94 \text{ N}$$

$$(f_b) \geq f_0$$

$$4635.94 \geq 779.50 \text{ design is safe}$$

$$F_d = F_t + \frac{21V (cb \cos^2 \beta + F_t) \cos \beta}{21V + \sqrt{cb \cos^2 \beta + F_t}}$$

$$F_t = P/v$$

$$= 15 \times 10^3 / 41.887 = 358.106 \text{ N}$$

$$C = \frac{K \cdot E_1 \cdot E_2 \cdot e}{(E_1 + E_2)}$$

$$= \frac{0.115 \times 2.15 \times 10^5 \times 1.1 \times 10^5 \times 0.025}{2.15 \times 10^5 + 1.1 \times 10^5}$$

$$= 209.21$$

$$F_d = 358.106 + \frac{21 \times 41.88 (209.21 \times 30 \cos^2 45 + 358.106) \cos 45}{21 \times 41.88 + \sqrt{209.21 \times 30 \cos^2 45 + 358.106}}$$

$$= 358.106 + \frac{2174637.027}{938.756}$$

$$= 2674.614 \text{ N}$$

→ wear strength

$$F_w = \frac{d_1 \cdot b \cdot Q \cdot K_w}{\cos^2 \beta}$$

$$Q = 1.6$$

$$K_w = \frac{H_e^2 \sin \alpha}{1.4} \left(\frac{1}{E_1} + \frac{1}{E_2} \right)$$

$$= \frac{618^2 \sin 20}{1.4} \left(\frac{1}{2.15 \times 10^5} + \frac{1}{1.1 \times 10^5} \right)$$

$$= 1.282$$

$$F_w = \frac{80 \times 30 \times 1.6 \times 1.282}{\cos^2 45}$$

$$F_w = 9845.76 \text{ N}$$

$$F_w \geq F_d$$

$$9845.76 \geq 2674.619$$

design is safe

→ other parameters

$$\text{tip circle dia} \rightarrow d_i + 2ha$$

$$80 + 2(3) = 86 \text{ mm}$$

$$\text{root circle dia} \rightarrow d_i - 2hf$$

$$= 80 - 2(3.75)$$

$$= 72.5 \text{ mm}$$

7. A pair of 20° full depth involute tooth spur gear is to transmit 30 kW at a speed 250 rpm of the pinion the transmission ratio is 4. The pinion is made up of cast steel having a allowable static stress 100 mpa while the gear is made up of C.2 having allowable static stress 55 mpa the pinion has 20 teeth & its face width 12.5 m. Determine the module, face width & pitch dia of both the pinion & gear from the stand point of strength by taking velocity factor due to consideration. The tooth factor is given by expression $y = 0.154 - \frac{0.912}{Z}$ & velocity factor is given by $C_v = \frac{3}{3+u}$

sd Given data

$$P = 30 \text{ kW}$$

$$N = 250 \text{ rpm}$$

$$i = 4 \rightarrow i = \frac{Z_2}{Z_1} = 4$$

$$(\sigma_b)_p = 100 \text{ mpa}$$

$$(\sigma_b)_g = 55 \text{ mpa}$$

$$z_p = 20 \text{ teeth}$$

$$b = 12.5 \text{ m}$$

$$y = 0.154 - \frac{0.912}{z}$$

$$y_p = 0.154 - \frac{0.912}{20} = 0.108$$

$$z_2/z_1 = 4$$

$$y_g = 0.154 - \frac{0.912}{4 \times 20} = 0.142 \checkmark$$

$$(\sigma_b)_p \times y_p = 100 \times 0.108 = 10.8$$

$$(\sigma_b)_g \times y_g = 55 \times 0.142 = 7.81 \checkmark$$

$$v = \frac{\pi d n}{60}$$

$$d_2 = m \cdot z_2$$

$$= \frac{\pi \times m \times 50 \times 250}{60000}$$

$$= 1.047 \text{ m m/s}$$

$$C_v = \frac{3}{3+v}$$

$$= \frac{3}{3+1.047 \text{ m}}$$

$$F_D = \frac{P \times K_s \times C_v}{v}$$

$$= \frac{30 \times 10^3 \times 1 \times \left(\frac{3}{3+1.047 \text{ m}} \right)}{1.047 \text{ m}}$$

$$= \frac{90,000}{3.14 \text{ m} + 1.09 \text{ m}^2}$$

$$F_b \geq F_D \text{ so equal both}$$

$$\frac{90000}{3.14m + 1.09m^2} = 55 \times \pi \times m \times 12.5m \times 0.142$$

$$\frac{90,000}{3.14m + 1.09m^2} = 306.69m^2$$

$$90,000 = 334.29m^4 + 963m^3$$

$$\rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-963 \pm \sqrt{963^2 - 4(334.29)(90,000)}}{2 \times 334.29}$$

$$= 15.02 \approx 16 \quad (\text{from data book preferred})$$

$$m = 16$$

(ii) face width $b = 12.5m$

$$= 12.5 \times 16 = 200mm$$

(iii) dia $d_1 = m \times z_1$

$$= 16 \times 20 = 320mm$$

$$d_2 = m \times z_2$$

$$= 16 \times 80 = 1280mm$$

8. A pair of helical gear 30° helix angle is used to transmit 15 kW at 10,000 rpm of the pinion. The velocity ratio is 4 both the gears are to be made by hardened steel of static strength 100 N/mm². The gears are 20 stub and pinion have 24 teeth the face width may be taken as 14 times the module. Find the module & face width from this stand point of strength & check the gears for wear.

Given data

$$P = 15 \text{ kW}$$

$$N = 10,000 \text{ rpm}$$

$$i = 4$$

$$(\sigma_b) = 100 \text{ N/mm}^2$$

$$(z)_p = 24$$

$$(d_1 = m z_1)$$

$$\begin{aligned} \rightarrow V &= \frac{\pi D N}{60} \\ &= \frac{\pi \times m \times 24 \times 10,000}{60,000} \\ &= 12.56 \text{ m/s} \quad (\because v \leq 20 \text{ m/s}) \end{aligned}$$

$$\begin{aligned} C_v &= \frac{G + V}{G} \\ &= \frac{G + 12.56 \text{ m}}{G} \end{aligned}$$

$$\begin{aligned} F_D &= \frac{P \times K_S \times C_v}{V} \\ &= \frac{15 \times 10^3 \times 1 \times \left(\frac{G + 12.56 \text{ m}}{G} \right)}{12.56 \text{ m}} \\ &= \frac{90,000 + 188400 \text{ m}}{75.36 \text{ m}} \end{aligned}$$

$F_b \geq F_D$ so equate both

$$\frac{90,000 + 188400 \text{ m}}{75.36 \text{ m}} = 100 \times \pi \times m \times 14 \text{ m} \times 0.152$$

$$z_v = \frac{z}{\cos^3 \beta} = \frac{24}{\cos^3 30} = 36.95$$

$$\begin{aligned} y_v &= 0.175 - \frac{0.912}{z} \\ &= 0.175 - \frac{0.912}{36.95} = 0.152 \end{aligned}$$

$$\frac{90,000 + 188400m}{75.36m} = 669.5 m^2$$

$$90,000 + 188400m = 50459.9 m^3$$

$$50459.9 m^3 - 188400m - 90,000 = 0$$

$$m = 2.13 \approx 2.5$$

$$m = \underline{\underline{2.5}}$$

$$b = 14m$$

$$= 14 \times 2.5 = \underline{\underline{35 mm}}$$

$$V = 12.56 \times 2.5 = \underline{\underline{31.41 m/s}}$$

$$F_t = P/V$$

$$= \frac{15 \times 10^3}{31.41} = 477.7 N$$

$$C = \frac{k \cdot e_1 \cdot e_2 \cdot e}{e_1 + e_2}$$

$$= \frac{0.115 \times 2.15 \times 10^5 \times 2.15 \times 10^5 \times 0.025}{(2.15 \times 10^5 + 2.15 \times 10^5)} = 309.06 N$$

$$F_d = 477.7 + \frac{21 \times 31.41 (309.06 \times 35 \cos^2 30 + 477.7) \cos 30}{21 \times 31.41 + \sqrt{309.06 \times 35 \cos^2 30 + 477.7}}$$

$$= 7000 N$$

$$Q = \frac{2i}{i+1} = \frac{2 \times 4}{4+1} = 1.6$$

$$F_w = \frac{d \cdot b \cdot Q \cdot k_w}{\cos^2 \beta} = \frac{60 \times 35 \times 1.6 \times 2.553}{\cos^2 30} = 11437.4 N$$

$$\underline{\underline{F_w}} \geq \underline{\underline{F_d}} \quad \& \quad \underline{\underline{F_b}} \geq \underline{\underline{F_o}}$$

$$11437.4 \geq 7000 N \quad \& \quad 4178.9 \geq 2977.70$$

design is safe

design is safe

* UNIT - 6 *

* Machine tool Elements *

* Levers :-

Introduction :- A lever is a rigid rod or bar Capable of turning about a fixed point called fulcrum. It is used as a machine to lift a load by the application of small effort. The ratio of load lifted to the effort applied is called "mechanical Advantage". Sometimes, a lever is merely used to facilitate the application of force in a desired direction. A lever may be "straight or curved" and the forces applied on the lever may be parallel (or) inclined to one another. The principle on which the lever works is same as that of moments.

Consider a straight lever with parallel forces acting in the same plane as shown in fig 15.1. The points "A" and "B" through which the load and effort is applied are known as load and effort points respectively. "F" is the fulcrum about which the lever is Capable of turning. The perpendicular distance between the load point and fulcrum (L_1) is known as "load arm" and the perpendicular distance between the effort point and fulcrum (L_2) is called "effort arm". According to the principle of moments

$$W \times l_1 = P \times l_2$$

(or)

$$\frac{W}{P} = \frac{l_2}{l_1}$$

i.e. Mechanical

Advantage,

$$M.A = \frac{W}{P} = \frac{l_2}{l_1}$$

* The ratio of effort arm and load arm i.e. l_2/l_1 is called "leverage".

A little Consideration will show that if a large load is to be lifted by a small effort, then the effort arm should be much greater than the load arm. In some Cases, it may possible to provide a lever with large effort arm due to space limitations. Therefore in order to obtain a great leverage "Compound levers" may be used. It may be made of straight pieces, which may be attached to one another with pin joints. The bell Cranked levers may be used instead of a number of jointed levers. In a Compound lever, the leverage is the product of leverage of various levers.

*, Application of levers in Engineering practice:

The load "W" and the effort "P" may be applied to the lever in three different ways as shown

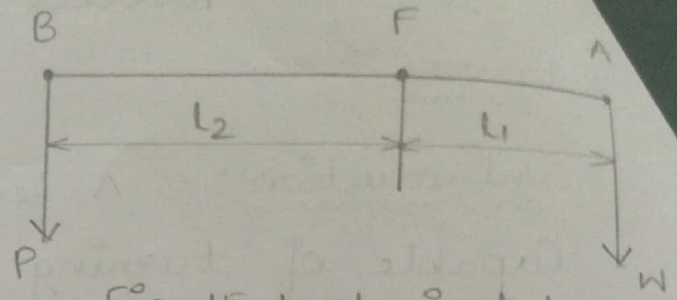
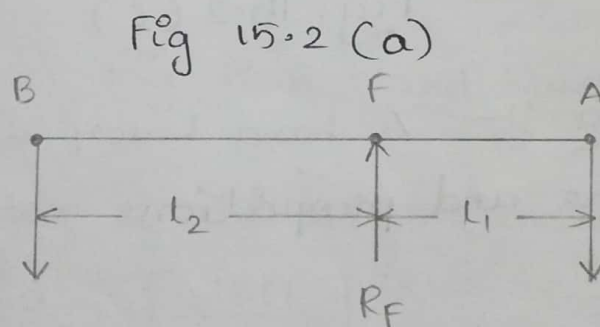


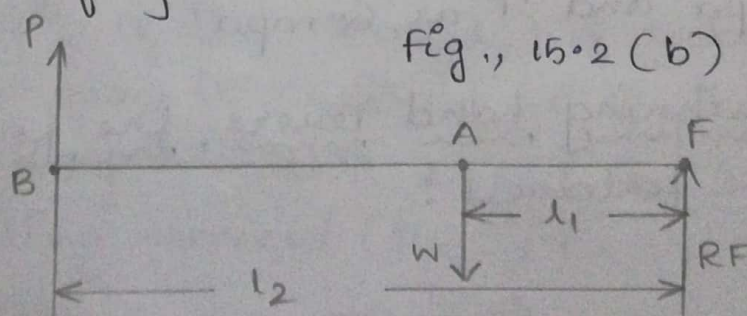
Fig. 15.1. Straight lever

in the fig 15.2. The levers shown at (a) (b) and (c) in fig 15.2 are called "first type", "second type" and "third type" of levers respectively

In the first type of levers, the fulcrum is in between the load and effort. In this Case, the effort arm is greater than load arm, therefore mechanical advantage obtained is more than One. Such type of levers are Commonly found in bell Cranked levers used in railway signalling arrangement, rocker arm in I.C. engines, handle of a hand pump, hand wheel of a punching press, beam of a balance, foot lever etc.



In the second type of levers, the load is in between the fulcrum and effort. In this Case, the effort arm is more than load arm, therefore the mechanical advantage is more than One. The application of such type of levers is found in levers of loaded safety valves.



In third type of levers, the effort is in between the fulcrum and load. Since the effort arm, in this case, is less than the load arm, therefore the mechanical advantage is less than One. The use of such type of levers is not recommended in engg practice. However a pair of tongs, the treadle of a sewing machine etc. are examples of this type of lever.

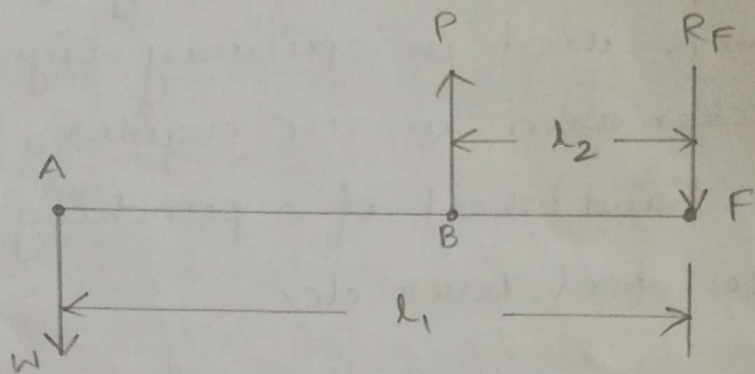


Fig. 15.2 (c)

Hand Lever :— A hand lever with suitable dimensions and proportions is shown in the fig. 15.8

Let P = force applied at the handle

L = Effective length of the lever

σ_t = permissible tensile stress, and

τ = permissible shear stress

*, for wrought iron, " σ_t " may be taken as "70mpa" and " τ " as "60mpa"

* In designing hand levers, the following procedure may be followed :

5) proportional dimensions of the key:-

$$l_k = h ; w_k = \frac{d}{4} ; t_k = \frac{d}{6} \text{ to } \frac{d}{4}$$

6) Tensile stress induced in the boss:-

$$\sigma_t = \frac{2FL}{ht_1(d+t_1)}$$

7) Bending stress induced in the lever near the boss:-

$$\sigma_b = \frac{6FL}{tB^2}$$

*, width of the lever $B = (4 \text{ to } 5)t$

8) Shear and Crushing stress induced in the key:-

$$\tau = \frac{2T}{l_k \cdot w_k \cdot d} ; \sigma_c = \frac{4T}{l_k \cdot t_k \cdot d}$$

$$*, T = F \times L , * , M = F \times L$$

$$*, T_e = \sqrt{M^2 + T^2}$$

Foot Lever :- A foot lever, as shown in the fig. 15.9 is similar to hand lever but in this case a foot plate is provided instead of handle. The foot lever may be designed in a similar way as discussed for hand lever. For foot lever, about 800N is considered

5) proportional dimensions of the key :-

$$l_k = h ; w_k = \frac{d}{4} ; t_k = \frac{d}{6} \text{ to } \frac{d}{4}$$

6) Tensile stress induced in the boss :-

$$\sigma_t = \frac{2FL}{nt_1(d+t_1)}$$

7) Bending stress induced in the lever near the boss :-

$$\sigma_b = \frac{6FL}{tB^2}$$

*, width of the lever $B = (4 \text{ to } 5) t$

8) Shear and Crushing stress induced in the key :-

$$\tau = \frac{2T}{l_k \cdot w_k \cdot d} ; \sigma_c = \frac{4T}{l_k \cdot t_k \cdot d}$$

$$*, T = F \times L, *, M = F \times L$$

$$*, T_e = \sqrt{M^2 + T^2}$$

Foot Lever :- A foot lever, as shown in the fig. 15.9 is similar to hand lever but in this case a foot plate is provided instead of handle. The foot lever may be designed in a similar way as discussed for hand lever. For foot lever, about 800N is considered

as full force which a man can exert in pushing a foot lever. The proportions of the foot plate as shown in fig., 15.9.

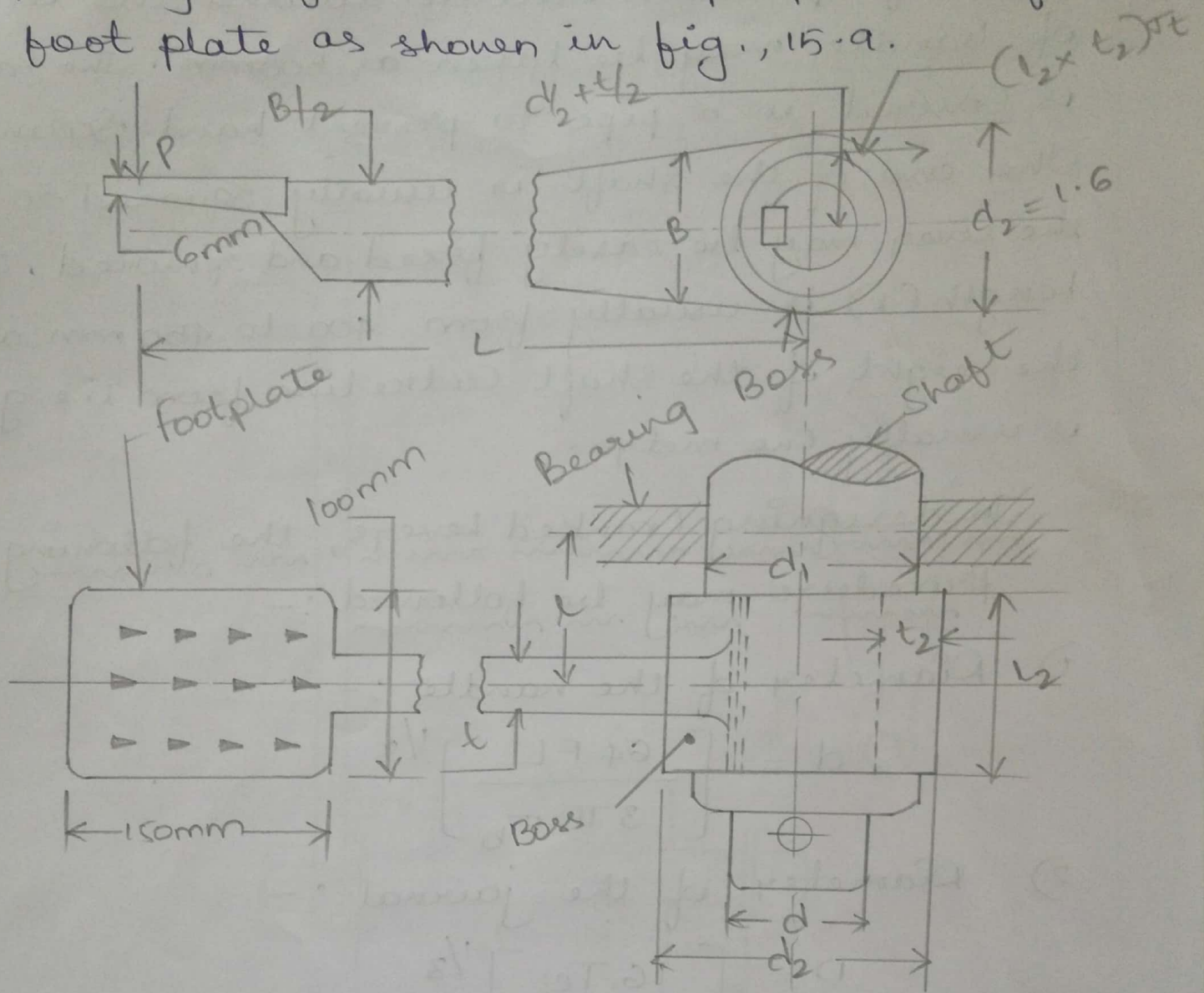


Fig. 15.9 * Foot Lever *

Cranked Lever : - A Cranked lever as shown in the figure . 15.10 , is a hand lever Commonly used for Operating hoisting winches.

The lever Can be Operated either by a single person (or) by two person . The max force in order to operate the lever may be taken as 400N and the length of the handle as 300mm . In Case the

Lever is operated by two persons, the max force of operation will be doubled and length of handle may be taken as 500mm. The handle is covered in a pipe to prevent hand scolding. The end of the shaft is usually squared so that the lever may be easily fixed and removed. The length (L) is usually from 400 to 450 mm and the height of the shaft centre line from the ground is usually one metre.

In designing Cranked Levers, the following procedure may be followed :-

1) Diameter of the handle :-

$$d = \left[\frac{64 FL}{3 \pi \cdot \sigma_b} \right]^{1/3}$$

2) Diameter of the journal :-

$$D = \left[\frac{16 T_e}{\pi \cdot \tau} \right]^{1/3}$$

$$\text{where } T_e = \sqrt{M^2 + T^2};$$

$$*, M = F \left(\frac{2}{3} l + x \right)$$

$$*, T = F \cdot l$$

3) Usual dimensions :-

$$\begin{aligned} d &= 25 \text{ mm for single person operation} \\ &= 40 \text{ mm for two person operation} \end{aligned}$$

→ Diametral Clearance for the Cover pipe (5)
 $= 3\text{mm (minimum)}$

*, $D = 30$ to 45 mm for single person

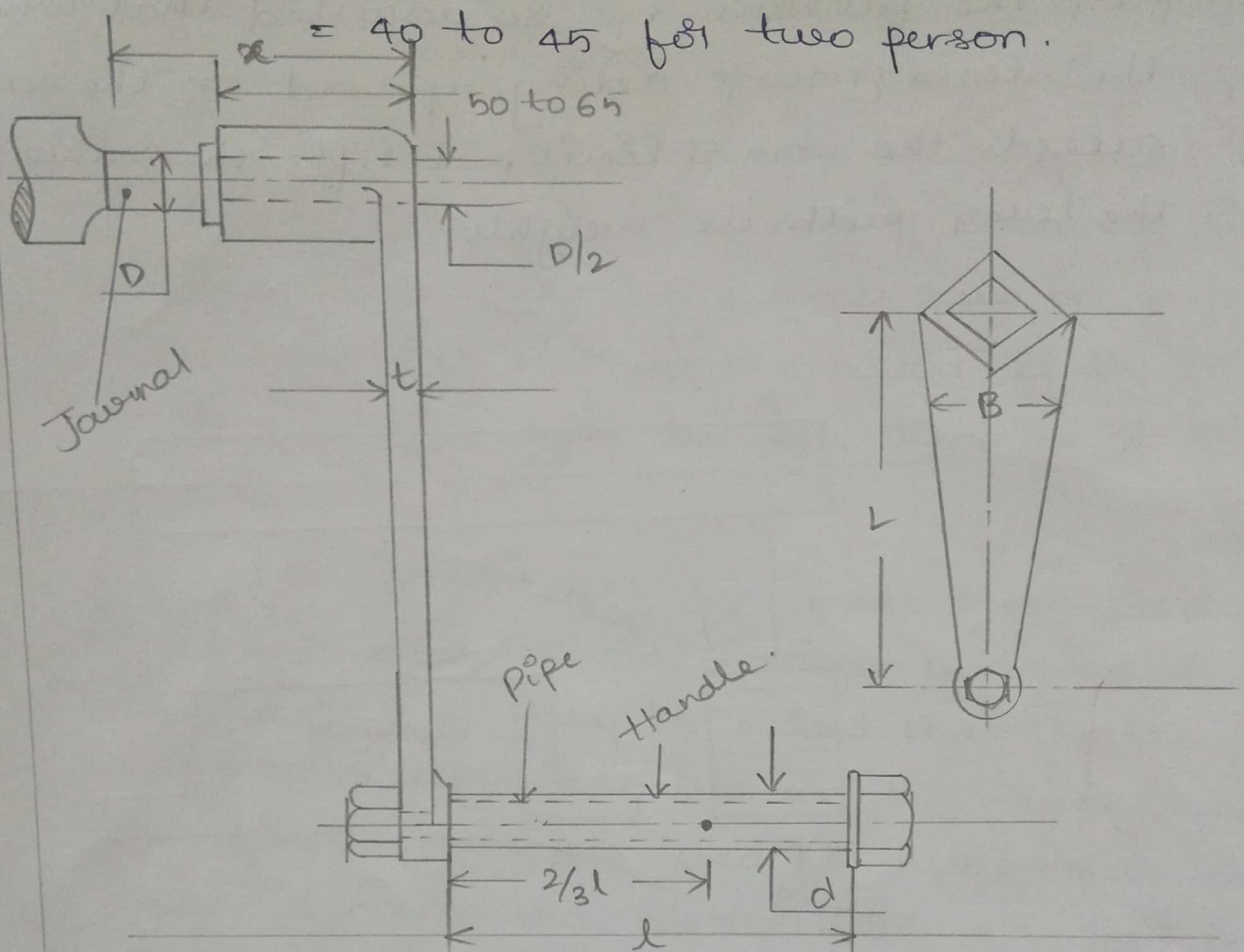


Fig. 15.10., Cranked Lever

Lever for a Lever Safety valve :- A lever for a lever safety valve is shown in fig. 15.11. It is used to maintain a constant safe pressure inside the boiler. When the pressure inside the boiler increases the safe value, the excess steam blows off through the valve automatically. The valve resets over the gunmetal seat which is secured to a casing fixed upon the boiler.

One end of the lever is pivoted at the fulcrum by a pin to the toggle, which the other end carries the weight. The weights and its distance from the fulcrum are so adjusted that when the steam pressure acting upward on the valve exceeds the normal limit, it lifts the valve & the lever with its weights.

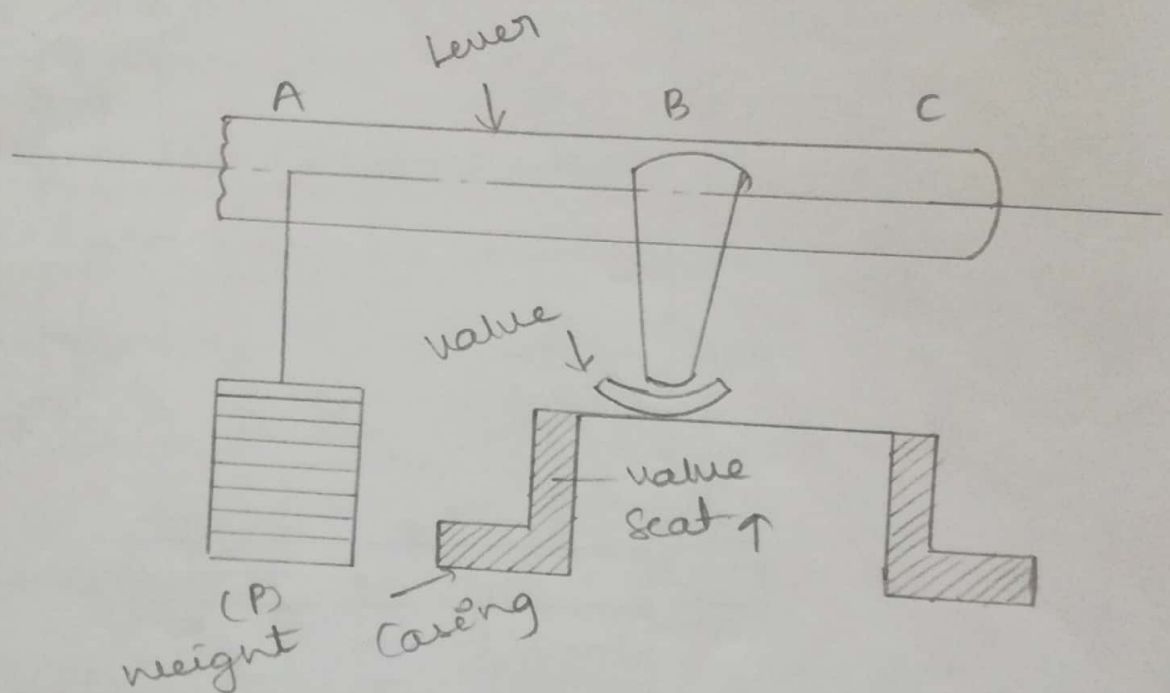


Fig. 15.11

Lever Safety valve

wire ropes : -

(6)

Introduction : - when a large amount of Power is to be transmitted over long distances from one pulley to another, then wire ropes are used. The wire ropes are widely used in the elevators, mine hoists, cranes, conveyors, and hauling devices. The wire ropes run on grooved pulleys but they rest on the bottom of the grooves and are not wedged between the side of the grooves.

The wire ropes are made from cold drawn in order to have increase the strength and durability. It may be noted that the strength of the wire ropes increases as its size decreases. The various materials used for wire ropes in order of increasing strength are wrought iron, cast steel, extra strong cast steel, plough steel & alloy steel. For certain purposes, the wire ropes may also be made of copper, bronze and stainless steels.

* Advantages of wire ropes : - The wire ropes have the following advantages as compared to fibre ropes.

- 1) These are lighter in weight.
- 2) These offer silent operation.

3. These Can withstand shock loads.
4. These are more reliable.
5. These are more durable.
6. They do not fail suddenly.
7. The efficiency is high and
8. The Cost is low.

*, Designation of wire Ropes:-

The wire ropes are designated by the number of wires in each strand. For example, a wire rope having six strands and seven wires in each strand is designated by 6x7 rope.

Following table shows the standard designation of ropes and their applications:

<u>Standard designation</u>	<u>Application</u>
6 x 7 Rope	It is standard Coarse laid rope used as haulage rope in mines, tramways, power transmission
6 x 19 Rope	It is standard hoisting rope used for hoisting purposes in mines, quarries, Cranes, dredges, elevators, tramways, well drilling
6 x 37 Rope	extra flexible hoisting rope used in steel mill ladder, Cranes, high speed elevators.
8 x 19 Rope	It is also an extra flexible Rope.

(*) procedure for designing a wire rope: (7)

1. first of all, select a suitable type of rope from tables (^{26.1,} 26.5, 26.6, 26.7) as per data book. for the given application.
2. find the design load by assuming a factor of safety 2 to 2.5 times the factor of safety given in table 20.11
3. find the diameter of the wire rope (d) by equating the tensile strength of the rope selected to the design load.
4. find the diameter of the wire (d_w) and area of rope (A) from the table 26.1 from Data book.
5. find the various stresses (or loads) in the rope.
6. find the effective stresses (or loads) during normal working, during starting and during acceleration of the load.
7. Now, find the actual factor of safety and compare with factor of safety given in . If the factor of safety is within permissible limits, then the design is safe.

Machine Tool Elements (lever & wire rope)

1. Hand lever of a brake is 0.8 m long from the center of gravity of the spindle to the point of applications of the pull of 300 N. The effective overhang from the nearest bearing is 100 mm of the permissible stress in tensile shear and crushing is not to exceed 66 N/mm². Design the spindle key and lever. Assume the arm of the lever rectangular having width twice of the thickness.

sol Given data

force applied at the handle $F = 300 \text{ N}$

effective length of the lever $L = 0.8 \text{ m} = 800 \text{ mm}$

effective overhang distance $l = 100 \text{ mm}$

allowable tensile stress
shear stress
crushing stress } = 66 \text{ N/mm}^2

→ Dia of shaft inside the lever

$$\begin{aligned} d &= \left[\frac{16T}{\pi \tau} \right]^{1/3} \\ &= \left(\frac{16 \times F \times L}{\pi \times 66} \right)^{1/3} \\ &= \left(\frac{16 \times (300 \times 800)}{\pi \times 66} \right)^{1/3} = 27 \text{ mm} \end{aligned}$$

→ dia of shaft inside the bearing

$$\begin{aligned} d_1 &= \left(\frac{16 T_e}{\pi \tau} \right)^{1/3} \\ &= \left(\frac{16 \times \sqrt{M^2 + T^2}}{\pi \times \tau} \right)^{1/3} \\ &= \left(\frac{16 \times \sqrt{(300 \times 100)^2 + (300 \times 800)^2}}{\pi \times 66} \right)^{1/3} \\ &= 27 \text{ mm} \end{aligned}$$

→ proportional dimensions of key

$$l_k = h$$

$$w_k = d/4$$

$$= 27/4 = 7 \text{ mm}$$

$$t_k = d/6$$

$$= 27/6 = 5 \text{ mm}$$

$$\tau = \frac{2T}{l_k \cdot w_k \cdot d}$$

$$l_k = \frac{2 \times (300 \times 800)}{66 \times 7 \times 27} = 38.48 \text{ mm}$$

$$\rightarrow t_1 = 0.3d$$

$$= 0.3 \times 27 = 8.1 \text{ mm}$$

$$\rightarrow d_s = 1.6 \times d$$

$$= 1.6 \times 27 = 43.2 \text{ mm}$$

→ Tensile stress

$$\sigma_t = \frac{2FL}{ht_1(d+t_1)}$$

$$= \frac{2 \times 300 \times 800}{39 \times 8.1 (27 + 8.1)} = 43.28 \text{ N/mm}^2$$

$$43.28 < 66 \text{ N/mm}^2 \text{ design is safe}$$

→ Bending stress

$$\sigma_b = \frac{6FL}{tB^2} \quad (B = 4t)$$

$$\sigma_b = \frac{6 \times FL}{t \times (4t)^2}$$

$$t^3 = \frac{GFL}{\sigma_b \times 16}$$

$$t = \sqrt[3]{\frac{6 \times 300 \times 800}{66 \times 16}}$$

$$t = 11.08 \text{ mm}$$

$$B = 4t$$

$$B = 4(11.08) = 44.32 \text{ mm}$$

→ width of the lever

$$b = B/2$$

$$= 44.32/2$$

$$= 22.16 \text{ mm}$$

The length of the handle grip may be taken as 150 mm with outer end of 27 mm diameter and gradually tapered to 25 mm dia at the inner end.

2. Design a foot brake lever from the following data, length of lever from the center of gravity of the spindle to be point of application of load 1m max load on the foot brake 800 N. overhang from the nearest bearing 100 mm. permissible tensile & shear stress 70 N/mm²

Given data :

$$\text{Effective length } L = 1\text{m} = 1000 \text{ mm}$$

$$\text{Load on foot brake } F = 800 \text{ N}$$

$$\text{overhang breaking } l = 100 \text{ mm}$$

$$\text{permissible tensile \& shear} = 70 \text{ N/mm}^2$$

→ dia of the shaft

$$d = \left[\frac{16T}{\pi \tau} \right]^{1/3}$$

$$d = \left[\frac{16 \times FL}{\pi \times \tau} \right]^{1/3}$$

$$= \left[\frac{16 \times (800 \times 1000)}{\pi \times 70} \right]^{1/3} = 38.75 \text{ mm}$$

→ dia of shaft inside the bearing

$$d_1 = \left[\frac{16 T_e}{\pi \times \tau} \right]^{1/3}$$

$$= \left[\frac{16 \times \sqrt{M^2 + T^2}}{\pi \times \tau} \right]^{1/3}$$

$$= \left[\frac{16 \times \sqrt{80000^2 + 800000^2}}{\pi \times 70} \right]^{1/3}$$

$$= 38.81 \approx 40 \text{ mm}$$

→ proportional dimensions of key

$$l_k = h$$

$$w_k = 1.25d$$

$$w_k = d/4$$

$$= 40/4 = 10 \text{ mm}$$

$$t_k = d/6$$

$$= 40/6 = 6.6 \approx 7 \text{ mm}$$

$$\tau = \frac{2T}{l_k \cdot w_k \cdot d}$$

$$l_k = \frac{2T}{\tau \cdot w_k \cdot d}$$

$$l_k = \frac{2 \times 800 \times 1000}{10 \times 70 \times 39}$$

$$= 59 \text{ mm}$$

$$\rightarrow \sigma_b = \frac{6 FL}{t B^2} \quad (\because B = 4t)$$

$$\sigma_b = \frac{6 FL}{t (4t)^2}$$

$$t^3 = \frac{6 FL}{16 \times \sigma_b}$$

$$t = \sqrt[3]{\frac{6 \times 800 \times 1000}{16 \times 70}}$$

$$t = 16.24 \text{ mm}$$

$$\rightarrow B = 4t$$

$$= 4 \times 16.24 = 65 \text{ mm}$$

$$\rightarrow b = B/2$$

$$= 65/2 = 32.5 \text{ mm}$$

$$\rightarrow t_1 = 0.3d$$

$$= 0.3 \times 40 = 12 \text{ mm}$$

$$\rightarrow \sigma_t = \frac{2 FL}{h t_1 (d + t_1)}$$

$$= \frac{2 \times 800 \times 1000}{59 \times 12 (40 + 12)}$$

$$= 43.45 \text{ N/mm}^2$$

The length of the foot plate may be taken as with outer end of 39 mm dia and the thickness of foot lever 17 mm and the width of the foot lever is handle 33 mm

3. Design a cranked lever for the following dimensions length of the handle 300 mm, length of the lever arm is 400 mm. overhang of the journal 100 mm. The lever is operated by a single person exerting a max force of 400 N at a distance of $\frac{1}{3}$ length of the handle from its force end. The permissible stress may be taken as 50 N/mm^2 in bending for lever & 40 N/mm^2 in shear for shaft.

sol Given data

length of the handle $l = 300 \text{ mm}$

length of the over arm $L = 400 \text{ mm}$

overhang of journal $x = 100 \text{ mm}$

max force $F = 400 \text{ N}$

$\sigma_b = 50 \text{ N/mm}^2$

$\tau_s = 40 \text{ N/mm}^2$

→ dia of the shaft handle

$$\begin{aligned} d &= \left[\frac{64 F l}{3 \pi \sigma_b} \right]^{1/3} \\ &= \left[\frac{64 \times 400 \times 300}{3 \times \pi \times 50} \right]^{1/3} \\ &= 25.35 \text{ mm} \end{aligned}$$

→ thickness of web

$$\sigma_b = \frac{7.5 F L}{t B^2} \quad (\because B = 2t)$$

$$\sigma_b = \frac{7.5 F L}{t (2t)^2}$$

$$t^3 = \frac{7.5 FL}{\sigma_b \times 4}$$

$$t = \sqrt[3]{\frac{7.5 \times 400 \times 400}{50 \times 4}}$$

$$t = 18.17 \text{ mm}$$

→ Dia of Journal

$$D = \left[\frac{16 T_e}{\pi \times \tau} \right]^{1/3}$$

$$= \left[\frac{16 \times 20,000}{\pi \times 40} \right]^{1/3}$$

$$= 29.42 \text{ mm}$$

4. Design a cranked lever for the following dimensions length of the handle 320 mm, length of the lever arm 450 mm, overhang of the journal 120 mm, the lever operated by a single person exerting a max force of 400 N. at a distance of $\frac{1}{3}$ length of the handle of the free end. 50 N/mm^2 for lever & 40 N/mm^2 for shaft

sol Given data :-

$$l = 320 \text{ mm}$$

$$L = 450 \text{ mm}$$

$$x = 120 \text{ mm}$$

$$F = 400 \text{ N}$$

$$\sigma_b = 50 \text{ N/mm}^2$$

$$\tau = 40 \text{ N/mm}^2$$

→ dia of the handle

$$d = \left[\frac{64 F \cdot l}{\pi \times 3 \times \sigma_b} \right]^{1/3}$$

$$= \left[\frac{64 \times 400 \times 320}{\pi \times 3 \times 50} \right]^{1/3} = 26 \text{ mm}$$

→ thickness of web

$$\sigma_b = \frac{7.5 FL}{t B^2} \quad (\because B = 2t)$$

$$\sigma_b = \frac{7.5 FL}{t \times (2t)^2}$$

$$t^3 = \frac{7.5 FL}{4 \times \sigma_b}$$

$$t = \sqrt[3]{\frac{7.5 \times 400 \times 450}{4 \times 50}}$$

$$t = 18.89 \text{ mm}$$

→ dia of the journal

$$\begin{aligned} D &= \left[\frac{16 T_e}{\pi \times T} \right]^{1/3} \\ &= \left[\frac{16 \times \sqrt{M^2 + T^2}}{\pi \times T} \right]^{1/3} \\ &= \left[\frac{16 \times 224003.96}{\pi \times 40} \right]^{1/3} \\ &= 30.55 \text{ mm} \end{aligned}$$

5. select a wire rope for a vertical mine hoist to lift a load of 90 kN from a depth of 500 m. A rope speed of 3 m/s is to be attain in 10 sec

sol Given data :

$$\text{load } P_r = 90 \times 10^3 \text{ N}$$

$$\text{depth } h = 500 \text{ m}$$

$$\text{speed } v = 3 \text{ m/s}$$

$$\text{time } t = 10 \text{ sec}$$

used on applications

6 x 19 is selected

$$dw = 0.07d$$

$$A = 0.4d^2$$

$$n = 10$$

$$k_d = 1.5$$

→ design load

$$\begin{aligned} P_d &= P_r \times n \times k_d \\ &= 30 \times 10^3 \times 10 \times 1.5 \\ &= 30 \times 10^4 \text{ N.} \end{aligned}$$

→ Net cross area of rope

$$\begin{aligned} A &= \frac{P_d}{\sigma_u} \\ &= \frac{30 \times 10^4}{1800} \\ &= 166.667 \end{aligned}$$

($\therefore \sigma_u = 1600 - 1900$)

→ dia of the rope

$$d = 1.5 dw \sqrt{i} \quad \text{⑧}$$

$$A = \frac{\pi}{4} d^2$$

$$d = \sqrt{A/0.4}$$

$$d = \sqrt{166.667/0.4}$$

$$d = 20.41$$

$$d = 22$$

$$w = 18.4$$

$$P = 284$$

} from table 26.6

Effective load acting on the rope

→ during normal working

$$F = F_t + F_b$$

$$F_t = P_r + (W \times l)$$

$$= 20 \times 10^3 + (18.4 \times 500)$$

$$= 29.200 \text{ KN}$$

$$F_b = A \cdot \frac{dw}{D_{min}} \cdot e'$$

$$= 0.4 \times (22)^2 \times \frac{0.07(22)}{100(22)} \times 0.8 \times 10^5$$

$$= 10.8416 \text{ KN}$$

$$F = 29.200 + 10.8416$$

$$= 40.104 \text{ KN}$$

$$F = P/n$$

$$n = P/F$$

$$= 284 / 40.104 = 7.09$$

$$F \leq P/n$$

$$40.104 \leq P/n \quad \text{design is safe}$$

→ during acceleration

$$F = F_t + F_b + F_a$$

$$F_a = F_t \cdot a/g \quad (\because a = v/t)$$

$$F_a = 29.200 \times \left(\frac{3/10}{9.81} \right)$$

$$= 0.89 \text{ KN}$$

$$F = 29.200 + 10.8416 + 0.89$$

$$= 40.934 \text{ KN}$$

$$F = P/n$$

$$\begin{aligned} n &= P/F \\ &= 284/40.934 \\ &= 6.93 \end{aligned}$$

$$F \leq P/n$$

40.934 $\leq$ P/n design is safe

→ During starting

$$F = F_{st} + F_b$$

$$\begin{aligned} F_{st} &= 2F_t \\ &= 2 \times 29.200 \\ &= 58.4 \text{ kN} \end{aligned}$$

$$\begin{aligned} F &= 58.4 + 10.8416 \\ &= 69.2416 \text{ kN} \end{aligned}$$

$$F = P/n$$

$$\begin{aligned} n &= P/F \\ &= 284/69.2416 \\ &= 4.1015 \end{aligned}$$

$$F \leq P/n$$

69.2416 $\leq$ P/n design is safe

6. Select a suitable wire rope by 6x37 group to lift a max load of 10 kN through a height of 60m. The weight of bucket is 2 kN. max lifting speed is 2 m/sec. which is attend in 3 sec the f.o.s is 6

Given data

$$\text{total weight} = 10 + 2 = 12 \text{ kN}$$

$$V = 2 \text{ m/s}$$

$$t = 3 \text{ sec}$$

$$h = 60 \text{ m}$$

6x37 group

→ from data book table no 26.1

$$\text{dia wire } d_w = 0.041 d$$

$$\text{Area } A = 0.4 d^2$$

→ from table no. 26.4

$$n = 6$$

$$K_d = 1.6$$

$$\text{Dim}/d = 22-27$$

→ Design load

$$P_d = P_t \times n \times K_d$$

$$= 12 \times 10^3 \times 6 \times 1.6$$

$$= 115.2 \text{ kN}$$

→ Net cross section

$$A = \frac{P_d}{\sigma_u}$$

$$= \frac{115.2 \times 10^3}{1800} = 64$$

→ dia of rope

$$d = \sqrt{A/0.4}$$

$$= \sqrt{64/0.4} = 12.6 \text{ mm} \approx 14 \text{ mm}$$

$$d = 14 \text{ mm}$$

$$W = 7.4$$

$$P = 113$$

} from table no 26.7

Effective load acting on the rope :

during normal working

$$F = F_t + F_b$$

$$\begin{aligned} F_t &= P_r + (w \times l) \\ &= 12 \times 10^3 + (7.4 \times 60) \\ &= 12.4 \times 10^3 \text{ N} \\ &= 12.4 \text{ kN} \end{aligned}$$

$$\begin{aligned} F_b &= A \cdot \frac{dw}{D_{\min}} \cdot E' \\ &= 64 \times \frac{0.041 (14)}{14 (25)} \times (0.8 \times 10^5) \\ &= 8.396 \text{ kN} \end{aligned}$$

$$\begin{aligned} F &= 12.4 + 8.396 \\ &= 20.7 \text{ kN} \end{aligned}$$

$$F = P/n$$

$$n = P/F$$

$$= 113/20.7 = 5.45$$

$$F \leq P/n$$

$$20.7 \leq P/n \text{ design is safe}$$

→ During acceleration of load

$$F = F_t + F_b + F_a$$

$$F_a = F_t (a/g)$$

$$(\because a = v/t)$$

$$= 12.4 \left(\frac{2/3}{9.81} \right)$$

$$= 0.8426 \text{ kN}$$

$$F = 12.4 + 8.396 + 0.8426$$

$$= 21.63 \text{ kN}$$

$$F = P/n$$

$$n = P/F$$

$$= 113 / 21.63 = 5.22$$

$$F \leq P/n$$

$$21.63 \leq P/n \text{ design is safe}$$

→ During starting

$$F = F_{st} + F_b$$

$$F_{st} = 2 F_t$$

$$= 2 \times 12.4 = 24.8 \text{ kN}$$

$$F = 24.8 + 8.396$$

$$= 33.196$$

$$F = P/n$$

$$n = P/F$$

$$= 113 / 33.196$$

$$= 3.40$$

$$F \leq P/n$$

$$33.196 \leq P/n \text{ design is safe}$$

8
 A foot lever is 1 m from the center of shaft to be point of application of 800 N load. Find the dia of shaft. Dimensions of key. Dimensions of rectangular arm of the foot lever at 60 mm from the center of shaft. Assume width of the arm is 3 times the thickness the allowable tensile stress may be taken as 73 mpa & the allowable shear stress as 70 mpa

Given data

Effective length $L = 1 \text{ m} = 1000 \text{ mm}$

load on foot brake $F = 800 \text{ N}$

$l = 60 \text{ mm}$

$B = 4t$

$\sigma_t = 73 \text{ mpa}$

$\tau = 70 \text{ mpa}$

$$\begin{aligned} \rightarrow \text{dia of shaft } d &= \left[\frac{16T}{\pi \tau} \right]^{1/3} \\ &= \left[\frac{16 \times (800 \times 1000)}{\pi \times 70} \right]^{1/3} = 38.75 \text{ mm} \end{aligned}$$

$\rightarrow$ dia of shaft inside the bearing

$$\begin{aligned} d_1 &= \left[\frac{16 T_e}{\pi \tau} \right]^{1/3} \\ &= \left[\frac{16 \times \sqrt{M^2 + T^2}}{\pi \times \tau} \right]^{1/3} \\ &= \left[\frac{16 \times \sqrt{(800 \times 60)^2 + (800 \times 1000)^2}}{\pi \times 70} \right]^{1/3} \\ &= \left[\frac{16 \times \sqrt{(48000)^2 + (800000)^2}}{\pi \times 70} \right]^{1/3} \\ &= 38.77 \approx 40 \text{ mm} \end{aligned}$$

→ proportional dimensions of key

$$l_k = h$$

$$w_k = d/4 \\ = 40/4 = 10 \text{ mm}$$

$$t_k = d/6 \\ = 40/6 = 6.67 \approx 7 \text{ mm}$$

$$\rightarrow T = \frac{2T}{l_k \cdot w_k \cdot d}$$

$$l_k = \frac{2 \times FL}{T \times w_k \times d}$$

$$l_k = \frac{2 \times (800 \times 1000)}{70 \times 10 \times 40} = 57.14 \text{ mm}$$

$$\rightarrow \sigma_b = \frac{6FL}{tB^2} \quad (\because B = 3t)$$

$$\sigma_b = \frac{6FL}{t(3t)^2}$$

$$t^3 = \frac{6 \times (800 \times 1000)}{73 \times 9}$$

$$t = 19.40 \text{ mm}$$

$$\rightarrow B = 4t \\ = 4 \times 19.40 = 58.21 \text{ mm}$$

$$\rightarrow b = B/2 \\ = 58.21/2 = 29.106 \text{ mm}$$

$$\rightarrow t_1 = 0.3d \\ = 0.3 \times 40 = 12 \text{ mm}$$

$$= \frac{2FL}{ht_1(d+t_1)}$$

$$= \frac{2 \times (800 \times 1000)}{57.14 \times 12 (40 + 12)}$$

$$= 44.88 \text{ N/mm}^2$$

The length of the foot plate may be taken as 150 mm with the dia 40 mm and the thickness of the foot lever is 19.40 mm and the width of the foot lever is handle 29.106 mm

* Safety valve - lever

Q. A lever loaded safety is 70 mm in dia and is to be design for a boiler to blow off at a pressure of 1 mpa. design a suitable mild steel lever of rectangular cross-section taking the allowable stress at 70 mpa in tension 50 mpa in shear and 25 mpa in bearing pressure. The pin is made of same material as that of the lever. The distance from the fulcrum to weight the lever is 380 mm and the distance b/w the fulcrum pin & pin connected to valve spindle links to the lever is 80 mm

sol Given data

dia of safety valve $D = 70 \text{ mm}$

allowable tensile (σ_t) = 70 N/mm²

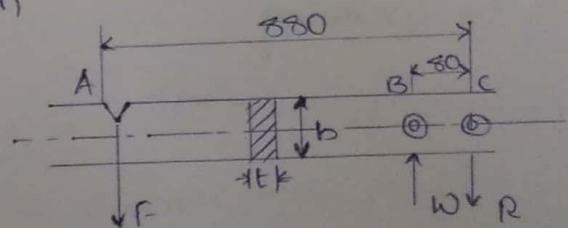
shear (τ) = 50 N/mm²

bearing (P_b) = 25 N/mm²

blow off pressure $P = 1 \text{ mpa} = 1 \text{ N/mm}^2$

The load applied on the lever at point B

$$W = \frac{\pi}{4} d^2 \cdot P$$



$$W = \frac{\pi}{4} (70)^2 \times 1$$

$$= 3.84 \times 10^3 \text{ N.}$$

for finding dead weight at point A. Take moments at point c

$$F \times 880 + R \times 0 = W \times 80$$

$$F \times 880 = W \times 80$$

$$F = \frac{3.84 \times 10^3 \times 80}{880}$$

$$F = 350 \text{ N}$$

$$R + F = W$$

$$R = W - F$$

$$R = 3.84 \times 10^3 - 350$$

$$R = 3490 \text{ N} \quad (\text{acting downward})$$

dimensions of pin at load point B & fulcrum point c.

d_b = dia of pin at point B

l_b = length of pin at point B

by assuming

$$l_b = 1.25 d_b$$

$$P_b = 25 \text{ N/mm}^2$$

$$P_b = \frac{W}{l_b \cdot d_b}$$

$$P_b = \frac{W}{d_b \times 1.25 d_b}$$

$$d_b^2 = \frac{3.84 \times 10^3}{25 \times 1.25}$$

$$d_b = 11.08 \text{ mm}$$

$$d_b = 1.25 d_b$$

$$= 1.25 \times 11.08 = 13.85 \approx 14 \text{ mm}$$

Then by following the above procedure the parameters l_c & d_c can be determined but since the load applied at point B & C are nearly equal we can take

$$d_c = 11.08 \text{ mm}$$

$$l_c = 14 \text{ mm}$$

checking the pin of shear strength :-

$$\tau = \frac{W}{2 \times \frac{\pi}{4} d_b^2}$$

$$= \frac{3.84 \times 10^3}{2 \times \frac{\pi}{4} (11.08)^2}$$

$$= 20.20 \text{ N/mm}^2 < 50 \text{ N/mm}^2 \quad \text{design is safe}$$

Dimensions of lever :-

The inner dia of the boss = pin dia + (2 x bush thickness)

$$= 11.08 + (2 \times 2) \quad (\because t = 2)$$

$$= 15.08 \text{ mm}$$

outer dia of the boss = 2 x pin dia

$$= 2 \times 11.08$$

$$= 22.08 \text{ mm}$$

t = thickness of lever

B = Width of lever.

$$\rightarrow \sigma_b = \frac{6 FL}{t B^2} \quad (\because B = 4t)$$

$$\sigma_b = \frac{6 FL}{t (4t)^2}$$

$$t_3 = \frac{6 \times 350 \times 880}{16 \times 70}$$

$$t = 11.81 \text{ mm}$$

$$B = 4t$$

$$= 4 \times 11.81$$

$$= 47.26 \text{ mm}$$

($\therefore$ assuming $f_b = f_t$).

9. A lever loaded safety valve is 75 mm in dia and is to be design for a boiler to blow off at a pressure of 1.1 mpa. Design a suitable mild steel lever of rectangular cross section using the following permissible stresses, tensile stress 72 mpa, shear stress 45 mpa, bearing pressure intensity 24 mpa. The pin is also made of mild steel the distance from the fulcrum to the weight of the lever is 900 mm and the distance b/w the fulcrum and pin connected the valve spindle links to the lever is 100 mm

Given data

dia of the safety valve $D = 75 \text{ mm}$

Blow off pressure $P = 1.1 \text{ mpa}$

tensile stress (f_t) = 72 mpa

shear stress (f_s) = 45 mpa

bearing pressure (P_b) = 24 mpa

length $L = 900 \text{ mm}$

length b/w B to C = 100 mm

The load applied on the lever at point B

$$W = \pi/4 d^2 \cdot P$$

$$= \frac{\pi}{4} (75)^2 \times 1.1$$

$$= 4859.65 \text{ N}$$

Finding the dead weight at point A. Take moment at point A.

$$F \times 900 = W \times 100$$

$$F = \frac{4859.65 \times 100}{900}$$

$$F = 539.96 \text{ N}$$

$$R + F = W$$

$$R = W - F$$

$$R = 4859.65 - 539.96 = 4319.68 \text{ N}$$

dimensions of pin at load point B & fulcrum point C :

d_b = dia of pin at point B

l_b = length of pin at point B

by assuming

$$l_b = 1.25 d_b ; P_b = 24 \text{ mpa}$$

$$P_b = \frac{W}{l_b \cdot d_b}$$

$$P_b = \frac{W}{1.25 d_b \times d_b}$$

$$d_b^2 = \frac{4859.65}{1.25 \times 24}$$

$$d_b = 12.72 \text{ mm} \approx 13 \text{ mm}$$

$$l_b = 1.25 d_b$$

$$= 1.25 \times 13$$

$$= 16 \text{ mm}$$

Then by following the above procedure the parameter l_c & d_c can be determined by since load applied at point B & C

are nearly equal to $d_c = 13 \text{ mm}$

$$l_c = 16 \text{ mm}$$

Checking the pin of shear strength :-

$$\tau = \frac{W}{2 \times \frac{\pi}{4} d b^2}$$

$$= \frac{4859.65}{2 \times \frac{\pi}{4} (13)^2} = 18.30 \text{ N/mm}^2 < 45 \text{ N/mm}^2$$

design is safe

Dimensions of lever :-

inner dia of bow = pin dia + (2 × bush thickness)

$$= 13 + (2 \times 2)$$

$$= 17 \text{ mm}$$

outer dia of bow = 2 × pin dia

$$= 2 \times 13 = 26 \text{ mm}$$

t = thickness of lever

B = width of lever

$$\rightarrow \sigma_b = \frac{6FL}{tB^2} \quad (\because B = 4t)$$

$$\sigma_b = \frac{6FL}{t(4t)^2}$$

(assuming $\sigma_b = \sigma_t$).

$$t^3 = \frac{6 \times 539.96 \times 900}{16 \times 72}$$

$$t = 13.62 \text{ mm} \approx 14 \text{ mm}$$

$$\rightarrow B = 4t$$

$$= 4 \times 13.62$$

$$= 54.48 \text{ mm}$$